

ARITHMETIC,

RATIONAL and PRACTICAL.

WHEREIN

The properties of NUMBERS are clearly pointed out, the THEORY of the science deduced from first principles, the methods of OPERATION demonstratively explained, and the whole reduced to PRACTICE in a great variety of useful RULES.

Consisting of THREE PARTS, viz.

- I. VULGAR ARITHMETIC.
- II. DECIMAL ARITHMETIC.
- III. PRACTICAL ARITHMETIC.

By JOHN MAIR, A.M.

PART I.

VULGAR ARITHMETIC.

EDINBURGH:

Printed by SANDS, MURRAY, and COCHRAN.
For A. KINCAID and J. BELL.

MDCCLXVI.

ARTHUR
RATIONAL and PRACTICAL

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Entered in Stationers Hall, according to act of parliament.



VULGAR ARITHMETIC

EDINBURGH

Printed by JAMES GUTHRIE and CO. for A. KILGUS and SONS

MDCCLXXVI

TO
THE RIGHT HONOURABLE
THOMAS EARL OF KINNOUL,

THE FOLLOWING
TREATISE OF ARITHMETIC,

IS,
WITH GREAT RESPECT,

INSCRIBED,

BY

HIS LORDSHIP'S

MUCH OBLIGED,

MOST OBEDIENT, AND

VERY HUMBLE SERVANT,

JOHN MAIR.

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P O A F E R

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P R E F A C E.

AN encomium on the usefulness and excellency of arithmetic would, in this place, be vain and idle. The world is already abundantly sensible of the worth and importance of this noble art. All, then, that seems necessary by way of preface, is to give some short account of the following treatise, and point out the improvements the reader may expect to meet with in perusing it.

And, in general, I persuade myself it will be found fully to answer the title-page; the properties of numbers being exhibited in a clear and instructive manner, and the whole theory built on first principles; that is, on axioms, or such propositions as are self-evident. The rules of operation, deduced immediately from the theory, are conceived in a few expressive words, and illustrated by a great variety of suitable examples. Care too has been taken to range things every where in such order, that what is prior paves the way for what is to follow.

By connecting things in this manner, the rugged path is rendered smooth, and the learner will proceed with pleasure from what is more simple, to what is more complex; and rise by easy steps from the lowest to every superior part of the science. And to furnish him with materials for exercise on every branch, and at the same time save labour to the teacher, sets of unwrought examples, with their answers, are inserted in all places where this appeared to be any way necessary, or of advantage. But to be more particular:

This treatise consists of three parts, under the titles of *Arithmetic Vulgar*, *Decimal*, and *Practical*. The first of these parts begins with an Introduction; wherein arithmetic is defined, its limits stated, and the reader taught to distinguish betwixt arithmetic, and the practical purposes to which its operations may be applied. After this, notation is explained at large, the various kinds of numbers

numbers described, and the axioms, or first principles, laid down, on which are founded the theory and rules of operation. Addition, subtraction, multiplication, and division, are taught at full length, with all the modern improvements, and the reason of the rules every where assigned.

The properties of proportional numbers, or numbers in geometrical proportion, are briefly pointed out; and thence is derived the rule of three: in which a simple and easy method of stating the questions is laid down; whereby the puzzling distinction of *direct* and *inverse* is precluded, or rendered needless, and the multiplicity of rules usually adhibited on this head, is avoided. Nor is this method confined to the simple rule of three; but extended to the rule of five, seven, nine, eleven, &c. The doctrine of vulgar fractions, and the rules of practice therewith connected, are delivered in a way simple and complete, without prolixity.

The second part contains decimal arithmetic, or the doctrine of decimal fractions. Here the origin of decimals of every kind, viz. finite, repeating, and circulating, is inquired into; their properties pointed out and described at large; the various sorts and ways of reduction are minutely explained, and properly exemplified: complete tables too are constructed for converting the parts of integers to decimals by inspection; and also another set of tables for reducing readily decimals of every sort to value. In addition, subtraction, multiplication, and division, the operations are conducted by a few plain easy rules; and these illustrated by a copious set of proper examples.

Some lovers of arithmetic have alledged, that authors, in treating of interminate decimals, are always curt and scanty in their explications, and leave this new part of decimal arithmetic so much involved in obscurity, as to be still, in their opinion, far above the reach of vulgar readers. But I flatter myself, that all complaints of this kind will now be removed; the methods of operation

in

in decimals of this sort being here set before the learner, with such clearness and perspicuity, that one, though of a low capacity, may soon acquire skill enough, not only to work examples wherein repeaters or circulates occur, but may even understand the reason of the process.

This second part concludes with a large decimal practice, setting forth the superior excellency of decimal arithmetic above every other method of numerical computation. Here is shewn the utility and conveniency of decimals in constructing tables of various kinds; how they may be used with great advantage in multiplication and division of duodecimals and sexagesimals; how they often rival vulgar fractions in point of accuracy, and surpass them in point of brevity; and, finally, that the decimal operation is frequently the shortest and easiest in the rule of three, the work being done that way in least time, and with fewest figures.

The third part contains the application of the other two to practical purposes, in a great variety of useful rules. Here facility is studied, the mercantile and fashionable methods of computation are introduced, and adapted to the several branches of commerce and business. Sometimes different ways of operation are exhibited, and the choice left to the practitioner. Large and correct tables on compound interest and annuities are inserted, with their construction and uses. Mensuration of surfaces and solids, artificers work, board and timber, &c. is taught in an easy manner, and at more than usual length.

To the above account I shall only add, that many things of importance, several small improvements, and some things curious, as well as useful, not hitherto taken notice of, will be found scattered here and there in different parts of the book. To conclude, no pains have been spared in collecting materials, and working them up, with a view to render this system of arithmetic instructive to the tyro, entertaining to the scholar, and useful to the man of business.

CON-

in addition to the four years just mentioned the following
with less than the full amount of the year's work
a few others, they have been in the service of the
to work a year or so, and in some cases a year or so
but may even be in the service of the year.

This second part contains a large amount of material for the study of the history of the country, and the history of the people, and the history of the government, and the history of the church, and the history of the world. It is a very interesting and valuable work, and is well worth a study.

than other works.

To the above account I shall only add that many things of importance, several small improvements, and some alterations as well as new, not subject to revision will be found scattered here and there in different parts of the work. To conclude, no paper have been issued in circulating pamphlets, and without it being with a view to render the Union of members more effective than at present existing in the world - and still to the state of union.

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ERRATA.

p. 39. In the Table of Long Measure, r. make 1 pole, perch, or rod.

p. 55. and 56. in the titles, instead of qrs. r. Q.

p. 74. in Ex 1. Method 2. r. the product 31392

p. 86. Rule VIII. l. 4 r. solid lines.

—— Rule IX. l. 4. r. solid inches.

p. 90. quest. 7. l. 9. for 36 r. 57 sq. inches.

p. 112. l. 1. r. Here

in. F. in.

p. 119. l. 10. r. C 295 = 24 7

p. 120. quest. 5. l. 5. r. 14697123087375.

p. 151. quest. 5. l. 1. r. 26 acres,

p. 167. quest. 25. l. 2. r. cost him in all L. 201 : 12 : 8, for 86 yards &c.

p. 232. for Note 1. r. Note.

p. 234. for Note 2 r. Note.

p. 238. Ex. 5. r. Prod. $3\frac{9}{16}$.

p. 240. l. penult. r. of the quot,

p. 241. l. 20. for multiplicand r. dividend

—— l. 26. r. $1 + 1 + 1 + \frac{2}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3} =$
 $\frac{11}{3}$ of $\frac{1}{4} = \frac{11}{12}$

p. 242. l. ult. r. Ans. $42\frac{7}{8}$.

p. 244 quest. 4. l. 2. for $3\frac{3}{4}$ r. $3\frac{9}{16}$.

p. 247. l. 20. r. the quots.

p. 260. Rule II. l. 2. r. divided into shillings, and some aliquot part or parts, divide it, &c.

p. 261. Rule III. l. 4. r. pound, or some number of shillings, and the subsequent, &c.

p. 270. in Ex. 2. on the margin for $\frac{1}{4}$ yd r. $\frac{1}{2}$ yd.

On the head of every left-hand page, instead of Chap. r. Part I.

Fellowship is now carried to Part III.

Directions to the BOOKBINDER.

Cancel the Title-page of volume 1. dated MDCCLXIV. with the Contents joined to it; and in place of these put the Title-page dated MDCCLXVI. with the Inscription, and Preface, and the Contents marked VOL. I. at the signature.

Cancel likewise the Title-page of volume 2. dated MDCCLXV. with the Errata joined to it; and in place of these put the Title-page dated MDCCLXVI. with the Contents, &c. joined to it.

Place the plate of Napier's rods to front p. 78. of vol. 1.

Place the four plates on Mensuration in successive order, at the end of vol. 3.; and sew them so as that every plate may, when opened out, be wholly without the book.

ARITHMETIC,

RATIONAL and PRACTICAL.

INTRODUCTION.

ARITHMETIC is a science explaining the properties of numbers, and shewing the method or art of computing by them.

The division of arithmetic into theory and practice is natural and just. The theory, or speculative part, treats of the nature and affections of numbers, and this is properly *science*. The practical part consists in the application of the rules gathered from the theory to the solution of questions or problems; and arithmetic in this respect is rightly termed an *art*.

Number, which is the object of arithmetic, is that which answers directly to the question, How many? and is either an unit, or some part or parts of an unit, or a multitude of units.

To a person having the idea of number in his mind the following questions naturally occur, *viz.* 1. How is such a number to be expressed or written? Hence we have Notation. 2. What is the sum of two or more numbers? Hence Addition. 3. What is the difference of two given numbers? Hence Subtraction. 4. What will be the result or product of a given number repeated or taken a certain number of times? Hence Multiplication. 5. How often is one given number contained in another? Hence Division.

These five, *viz.* Notation, Addition, Subtraction, Multiplication, and Division, are the chief parts, or rather

ther the whole of arithmetic. Every arithmetical operation requires the use of some of these, and nothing but a proper mixture of them is necessary in any operation whatever; and, by an Arabic term, these are called the *algorithm*.

These may be fitly compared to the five mechanic powers, which, variously combined, produce engines of different forms, and of amazing force; and which, at first sight, are apt to strike us with surprise; but upon examination, their effects are all found to flow from a proper combination of the simple powers. In like manner, we meet with a great variety of complex arithmetical operations, curious calculi, and astonishing solutions, which yet are all effected by a due application of the different parts of the algorithm.

Numerical computations are necessary almost in every affair; but the manner of the operations depends upon the nature of the subject to which they are applied. Arithmetic is a handmaid to most of her sister arts and sciences; and while she is employed in their service, she must be under their direction, and observe the laws they prescribe. Thus the calculation of a solar or lunar eclipse is performed by numbers; but the operation is directed by the precepts of astronomy: a reckoning at sea is wrought by numbers; but the steps of the work are conducted by the rules of navigation: the contents of a cask is cast up by numbers; but the art of gauging guides the operation: the perpendicular height of a hill is computed by numbers; but it is trigonometry that directs the manner of their application.

The canons directing the application of numbers in other sciences are not to be reckoned arithmetical rules: no rule is to be esteemed such, but where the operation entirely depends on arithmetical principles, and proceeds without any foreign aid or direction.

Hence from the number of arithmetical rules is excluded the rule of Three, with its numerous train of dependents, such as, Fellowship, the rule of False, Interest, Annuities, Alligation, Exchange, Barter, &c. The doctrine of proportion is delivered in the elements of geometry.

metry. It is Euclid, *lib. 6. prop. 16.* who demonstrates, that if four quantities be proportional, the rectangle, or product of the extremes, will be equal to that of the means: it is this property of proportionals that gives birth to the rule of Three; it is this that directs the arithmetical operation. For the like reason, Involution and Extraction of roots are no arithmetical rules; it is algebra that teaches the manner of these operations. Reduction and the rules of Practice are likewise to be exterminated; as also, addition, subtraction, &c. of the parts of integers, such as, shillings, pence, farthings, ounces, &c. The statutes of a country, with respect to coins, weights, and measures, prescribe the method of operation on these heads. It remains, then, that the five parts of which the algorithm consists make the whole of arithmetic: here the limits of this science are to be fixed, otherwise it would prove endless and inexhaustible.

True it is, that writers on this subject go generally beyond these bounds: they frequently introduce into their printed treatises all the rules excluded above, and several others. Nor is this any fault: authors are at liberty, and in the right, to store their writings with all such numerical operations as may be useful to the public. The only thing blameable in their conduct is, their not stating the limits of the science, and teaching their readers to distinguish betwixt arithmetic, and the application of its operations to practical or useful subjects.

The neglect here taken notice of has been attended with this remarkable consequence, that vulgar readers have generally mistaken these adventitious rules for parts of arithmetic; and as they observed new rules published every now and then in new books, they were apt to imagine that arithmetic admitted of no limits, and could never be attained to any sort of perfection.

Arithmetic, as a science, ought to demonstrate her own rules; that is, the rules of operation in addition, subtraction, multiplication, and division, both of integers and fractions; but this is all: to employ her in demonstrating the rules borrowed from other arts and sciences, would be vain and fruitless. An attempt of

this kind would be making her act out of sphere, and putting her upon an office she is by no means fit for. Numerical demonstrations are successfully applied to subjects purely arithmetical, and they can be carried little further. The proper demonstrations of these foreign rules are only to be had from geometry, algebra, or some other branch of science; and as these are much beyond the reach of the young arithmetician, he cannot at present expect entire satisfaction on this head, but must wait with patience, till, in the course of his studies, he come to be acquainted with the superior sciences.

The following treatise is designed as a rational, and at the same time a practical system of arithmetic; wherein the five parts of this science are fully but succinctly handled, and all the arithmetical rules numerically demonstrated. The adventitious rules depending on other sciences are likewise explained, and exemplified in the way that appears best for answering the purposes of the merchant, the accomptant, and mechanic. Some few things too are introduced for the entertainment of the scholar and the curious.

N O T A T I O N.

NOTATION is that part of arithmetic which explains the method of writing down, by characters or symbols, any number expressed in words; as also the way of reading or expressing, in words, any number given in characters or symbols. But the first of these is properly notation, and the last is more usually called *numeration*.

The ancient Romans, for this purpose, made choice of a few of the letters of their alphabet; and this is called the *literal*, or *Roman notation*. This way is still of considerable use, and necessary to be understood. But the method which now prevails, and is practised over a great part of the known world, is effected by characters or symbols, called *figures*; and is named *figural notation*, or *notation of numbers by figures*.

The things then proper to be comprised in this chapter are, 1. The literal or Roman notation. 2. The figural notation. 3. Numeration, or the way of reading numbers. 4. Descriptions of the kinds or species of numbers. 5. Arithmetical principles, or axioms. 6. The signification of such characters as are sometimes used for brevity's sake. 7. Some practical questions.

I. *The Roman Notation.*

The letters used by the ancient Romans, in their notation of numbers, were only these five, C, I, L, V, X.; whose values and order are as follows.

One, Five, Ten, Fifty, An hundred, Five hundred, A thousand.

I, V, X, L, C, D, CIO

By the repetition or combination of these they expressed any other number, according to the following rules.

I. The repetition of a letter repeats its value.

Thus II signifies two, III three; and XX twenty,
XXX

XXX thirty; and CC two hundred, CCC three hundred, &c.

II. The annexing a letter of a lower value to a letter of a higher value, adds their values, or denotes their sum.

Thus, VI signifies six, VII seven, VIII eight; and XI denotes eleven, XII twelve, XV fifteen, XVI sixteen; and LV fifty-five, LX sixty, LXX seventy, LXXX eighty; and CV one hundred and five, CX one hundred and ten, CL one hundred and fifty, CLV one hundred and fifty five; and IDC six hundred, IDCC seven hundred; and CIO, ID one thousand five hundred, CIO, IDC one thousand six hundred, CIO, IDCC one thousand seven hundred, &c.

III. The prefixing a letter of a lower value to a letter of a higher value, subtracts their values, or denotes their difference.

Thus, IV signifies four, IX nine, XL forty, XC ninety, IC ninety-nine, &c.

IV. The annexing of I to the number ID makes its value ten times greater.

Thus, IDD signifies five thousand, and IDDD fifty thousand.

V. The prefixing of C, together with the annexing of I, to the number CIO, makes its value ten times greater.

Thus, CCIDD denotes ten thousand, and CCCIDD an hundred thousand.

The ancients, as Pliny observes, carried this kind of notation no higher. If at any time they had occasion to express a greater number, they did it by repetition; thus, CCCIDD, CCCIDD signified two hundred thousand, and CCCIDD, CCCIDD, CCCIDD denoted three hundred thousand, &c.

The Romans, in later times, instead of ID used D; and instead of CIO they introduced M.

They likewise sometimes expressed thousands by a line drawn over the head of numbers. Thus, $\overline{\text{III}}$ signifies three

three thousand, $\overline{V}$ five thousand, $\overline{X}$ ten thousand, $\overline{LX}$ sixty thousand, $\overline{C}$ an hundred thousand, $\overline{M}$ a thousand thousands, or a million, $\overline{MM}$ two millions, &c.

In some editions of Cæsar's commentaries we find ∞ used, instead of CID , to signify a thousand.

Any thing further necessary to be known in the Roman notation may be learned from the following table; in which, with a view to exercise the learner, as well as for the sake of brevity, I have chosen to express the value of the literal numbers by figures, rather than words.

T A B L E.

I.	—	—	1	XC.	—	—	—	90
II.	—	—	2	C.	—	—	—	100
III.	—	—	3	CI.	—	—	—	101
IV. or IIII.	—	—	4	CII, &c.	—	—	—	102
V.	—	—	5	CC.	—	—	—	200
VI.	—	—	6	CCC.	—	—	—	300
VII.	—	—	7	CCCC. or CD.	—	—	—	400
VIII. or IIX.	—	—	8	IC. or D.	—	—	—	500
IX.	—	—	9	IDC. or DC.	—	—	—	600
X.	—	—	10	IDCC. or DCC.	—	—	—	700
XI.	—	—	11	IDCCC. or DCCC	—	—	—	800
XII.	—	—	12	IDCCCC. or DCCCC. or CM.	—	—	—	900
XIII.	—	—	13	CID. or M.	—	—	—	1,000
XIV. or XIIII	—	—	14	CID.C. or MC.	—	—	—	1,100
XV.	—	—	15	CID,CC or MCC. &c.	—	—	—	1,200
XVI.	—	—	16	MM. or II.	—	—	—	2,000
XVII.	—	—	17	MMM. or III	—	—	—	3,000
XVIII.	—	—	18	MMMM or IV.	—	—	—	4,000
XIX.	—	—	19	IDC or V.	—	—	—	5,000
XX.	—	—	20	IDCM. or VI.	—	—	—	6,000
XXI.	—	—	21	IDCMM. or VII.	—	—	—	7,000
XXII.	—	—	22	IDCMMM. or VIII.	—	—	—	8,000
XXIII.	—	—	23	IDCMMM or IX.	—	—	—	9,000
XXIV. or XXIIII.	—	—	24	CCIDC. or X.	—	—	—	10,000
XXV.	—	—	25	CCIDCM. or XI.	—	—	—	11,000
XXVI.	—	—	26	CCIDCMM. or XII. &c.	—	—	—	12,000
XXVII.	—	—	27	IDCC. or L.	—	—	—	50,000
XXVIII.	—	—	28	IDCCM. or LI.	—	—	—	51,000
XXIX.	—	—	29	IDCCMM. or LII. &c.	—	—	—	52,000
XXX.	—	—	30	CCCIDCC. or C.	—	—	—	100,000
XL.	—	—	40	CID,ICC,LXIII. or M,DCC,LXIII.	—	—	—	1763
L.	—	—	50	II. Figural	—	—	—	
LX.	—	—	60		—	—	—	
LXX.	—	—	70		—	—	—	
LXXX.	—	—	80		—	—	—	

II. *Figural Notation.*

An unit, or unity, is that number by which any thing is called one of its kind. It is the first number; and if to it be added another unit, we shall have another number, called *two*; and if to this last another unit be added, we shall have another number, called *three*; and thus, by the continual addition of an unit, there will arise an infinite increase of numbers. On the other hand, if from unity any part be subtracted, and again from that part another part be taken away, and this be done continually, we shall have an infinite decrease of numbers.

But though number, with respect to increase and decrease, be infinite, and knows no limits; yet ten figures, variously combined or repeated, are found sufficient to express any number whatsoever. These, with the method of notation by them, were originally invented by some of the eastern nations, probably the Indians; afterwards improved by the Arabians; and at last brought over to Europe, particularly into Britain, betwixt the tenth and twelfth century. From the ten fingers of the hands, on which it had been usual to compute numbers, the figures were called *digits*. Their form, order, and value, are as follows.

1 one, an unit, or unity, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 cipher, nought, null, or nothing. Of these, the first nine, in contradistinction to the cipher, are called *significant figures*.

The value of the figures now assigned is called their *simple value*, as being that which they have in themselves, or when they stand alone. But when two or more figures are joined as in a line, the figures then receive also a local value from the place in which they stand, reckoning the order of places from the right hand towards the left, thus,

~ First

7	Twelfth place.
7	Eleventh place.
7	Tenth place.
7	Ninth place.
7	Eighth place.
7	Seventh place.
7	Sixth place.
7	Fifth place.
7	Fourth place.
7	Third place.
7	Second place.
7	First place.

A figure standing in the first place has only its simple value; but a figure in the second place has ten times the value it would have in the first place; and a figure in the third place has ten times the value it would have in the second place; and universally a figure in any superior place has ten times the value it would have in the next inferior place.

Hence it is plain, that a figure in the first place simply signifies so many units as the figure expresses; but the same figure advanced to the second place will signify so many tens; in the third place, it will signify so many hundreds; in the fourth place, so many thousands; in the fifth place, so many ten thousands; in the sixth place, so many hundred thousands; and in the seventh place, so many millions, &c. Thus, 7 in the first place, will denote seven units; in the second place, seven tens, or seventy; in the third place, seven hundred; in the fourth place, seven thousand, &c.

Every three places, reckoning from the right hand, make a half-period; and the right-hand figures of these half-periods are termed *units* and *thousands* by turns; the middle figure is always tens, and the left-hand figure always hundreds. But here observe, that the right-hand figure of every half-period, properly and strictly speaking, is always units; for the place called *thousands*, when expressed at more length, is termed *units of thousands*, or the *units place of thousands*.

Two half-periods, or six places, make a full period; and the periods, reckoning from the right hand towards the left, are titled as follows, *viz.* the first is the period of *units*; the second, that of *millions*; the third is titled *bimillions*, or *billions*; the fourth, *trimillions*, or *trillions*;

the fifth, *quadrillions*; the sixth, *quintillions*; the seventh, *sextillions*; the eighth, *septillions*; the ninth, *octillions*; the tenth, *nonillions*, &c.

Half-periods are usually distinguished from one another by a comma, and full periods by a point or colon; as in the table following.

T A B L E.

4th Period.			3d Period.			2d Period.			1st Period.		
Trillions.			Billions.			Millions.			Units.		
Hundred thousands.	Ten thousands.	Thousands.	Hundred thousands.	Ten thousands.	Thousands.	Hundred thousands.	Ten thousands.	Thousands.	Hundred thousands.	Ten thousands.	Thousands.
Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.
9 6 4, 0 8 5	8 1 3, 7 0 0	2 3 7, 8 9 4	6 7 8, 0 4 0								

The table may be expressed in a more concise form thus,

4.	3.	2.	1. Per.
Trillions.	Billions.	Millions.	Units.
$\overline{C} \overline{X} M, C X U$	$\overline{C} \overline{X} M, C X U$	$\overline{C} \overline{X} M, C X U$	$\overline{C} \overline{X} M, C X U$
9 6 4, 0 8 5	8 1 3, 7 0 0	2 3 7, 8 9 4	6 7 8, 0 4 0

From the table it is obvious, that though a cipher signify nothing of itself, yet it serves to supply vacant places, and raises the value of significant figures on its left hand, by throwing them into higher places. Thus, in the first period, by a cipher's filling the place of units, the figure 4 is thrown into the place of tens, and signifies forty. And a cipher likewise supplying the place of hundreds, the figure 8 is advanced to the next higher place, and signifies eight thousand, &c. But a cipher does not change the value of a significant figure on its right hand. Thus, 07, or 007, is the same as 7.

From the table it likewise appears how any number expressed

expressed in words, may be written down by figures; in the doing of which observe the following

R U L E.

Beginning at the left hand, and writing towards the right, put every figure in such place and period as the verbal expression points out, supplying the omitted places with ciphers. Examples follow.

	Millions.	Units.
	CXM, CXU:	CXM, CXU:
Nine hundred and eighty-seven millions	987	000,000
Forty-five millions and seven hundred thousand	45	700,000
Six millions four hundred and thirty-two thousand	6	432,000
Eight hundred and five thousand and nine hundred		805,900
Seventy-three thousand and ten		73,010
Five thousand and four		5,004
Four hundred and twenty		420
Ninety-five		95
Seven		7

is written thus,

III. Numeration.

Notation and numeration are so nearly allied, that he who understands the former cannot fail soon to acquire the latter. The method of reading numbers when expressed by letters, is sufficiently clear from the table subjoined to the Roman notation; and the way of reading any number expressed by figures, may be easily learned from the table of the figural notation; in the doing of which observe the following

R U L E.

Beginning at the left hand, and reading toward the right; to the simple value of every figure join the name of its place, and conclude each period by expressing its title, every where omitting the ciphers. Examples follow.

B 2

Millions.

Millions.	Units.	
CXM, CXU :	CXM, CXU.	
900:000,000		} is read thus.
78:009,000		
4:000,800		
903,005		
20,034		
8,469		
708		
96		
3		
		Nine hundred millions.
		Seventy-eight millions and nine thousand.
		Four millions and eight hundred.
		Nine hundred and three thousand and five.
		Twenty thousand and thirty-four.
		Eight thousand four hundred and sixty-nine.
		Seven hundred and eight.
		Ninety-six.
		Three, or Three units.

N. B. The name of the unit's place, in all periods, and the title of the right-hand period, are commonly omitted, or very rarely expressed.

IV. Descriptions of the kinds or species of numbers.

The species of numbers are very various and manifold; but in this place it will be sufficient to describe such as appear most useful and necessary; particularly those that will occur in the ensuing treatise.

1. An *integer*, or *whole number*, is an unit, or any multitude of units; as 1, 7, 48, 100, 125.

2. A *fraction*, or *broken number*, is any part or parts of an unit; and is expressed by two numbers, which are separated from one another by a line drawn betwixt them; the under number being called the *denominator*, and the upper one the *numerator*, of the fraction; as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{9}{10}$.

3. A *mixt number* is an integer with a fraction joined to it; as $4\frac{1}{3}$, $7\frac{3}{4}$, $48\frac{5}{6}$.

4. A number is said to *measure* another number, when it is contained in that other number a certain number of times, or when it divides that other number without any remainder. Thus, 3 measures 6, 9, or 12.

5. An *even number* is that which is measured by 2, or which 2 divides without any remainder; as 2, 4, 6, 8, 10, 12.

6. An *odd number* is that which 2 does not measure, or which cannot be divided by 2, without a remainder; as 1, 3, 5, 7, 9, 11, 13.

7. A *prime number* is that which unity, or itself, only measures; as 3, 5, 7, 11, 13, 17, 19.

8. A *composite number* is that which is measured by some other number than itself, or unity; as 12, which is measured by 2, 3, 4, or 6.

9. Numbers are called *prime* to one another, when unity only measures them. Thus 13 and 36 are prime to one another; for no number, except unity, measures both.

10. Numbers are called *composite* to one another, when some number, besides unity, measures them. Thus 12 and 18 are composite to one another; for 3 or 6 measures both of them.

11. A number which measures another is called an *aliquot part* of that other. Thus 6 is an *aliquot part* of 18, and 3 of 12, and 5 of 20.

12. The number measured, or which contains the *aliquot part* a certain number of times, is called a *multiple* of that *aliquot part*. Thus 18 is a multiple of 6, and 12 of 3.

13. A number is called an *aliquant part* of another, when it does not divide that other without a remainder. Thus 7 is an *aliquant part* of 24.

14. Two, three, or more numbers, which, multiplied together, produce another number, are called the *component parts* of the number produced. Thus 3 and 4, or 2 and 6, are the *component parts* of 12; and 2, 3, and 4, are the component parts of 24.

15. The product of a number multiplied into itself is called the *square*, or *second power*, of that number; and the number itself is in this case called the *root*. And if the square be multiplied into the root, the product is called the *cube*, or *third power*, of that number. And if the cube be multiplied into the root, the product thence arising is called the *biquadrate*, or *fourth power*, &c.

V. *Arithmetical Principles or Axioms.*

An axiom or first principle in any science is a self-evident proposition which needs no demonstration. The following

following ones flow necessarily from the nature of numbers, the manner of their notation, or their specific properties. On them is founded the theory of arithmetic, and from them are deduced the rules to be observed in every operation.

I. Any given number may be increased or diminished at pleasure.

For there is no number so great, but a greater may be given; nor is there any number so small, but a smaller may be assigned.

II. The figures of any number increase in value from the right toward the left hand.

Hence in addition, subtraction, and multiplication, the operation begins at the units figure or lowest place on the right hand, and proceeds, with the flux of numbers, to the highest place on the left.

III. Ten in any inferior place makes one or an unit in the next higher place, and the reverse.

Hence when the sum of any column added, or the product of any two figures multiplied, amounts to or exceeds ten, we carry an unit for every ten to the next higher place: for the highest digit being 9, any number above 9 is compound, and requires more than one place to express it; which is done by removing or carrying away the tens, as so many units, to the next superior place.

IV. If the right-hand figure of any number be cut off, the remaining figure or figures are a just number of tens, and the right-hand figure so cut off is the overplus.

Hence in addition and multiplication, the right-hand figure of the sum of any column, or of the product of any two figures, is always set down, and the remaining figure or figures are carried on to the next higher place. Thus if a sum or product amount to 10, the 0 is set down, and 1 is carried; if it amount to 72, the 2 is set down, and 7 is carried; if it amount to 100, then 0 is set down, and 10 is carried; if it amount to 136, then 6 is set down, and 13 carried, &c.: but if the sum or product amount only to a single digit, then the digit is set down, and nothing is carried.

V. A number, by having one, two, three, &c. ciphers

phers annexed to it, becomes ten, a hundred, a thousand, &c. times greater.

Because the figures of the number are, by this means, thrown into higher places, thus 7 by a cipher annexed becomes 70; by two ciphers 700, and by three ciphers 7000, &c.

VI. Any number is naturally resolved into as many constituent parts as it has significant figures, by annexing to each significant figure as many ciphers as there are figures on its right hand.

Thus 345 is resolved into 300, 40, and 5; and 6082 is resolved into 6000, 80, and 2; and 70090 into 70000 and 90.

VII. The separate value of a single figure in any number is greater by one at least, than the value of all the figures on its right hand.

Thus, in the number 199, the separate value of 1, viz. 100, is greater by 1 than the figures on its right hand 99.

VIII. An unit is an aliquot part of every whole number, and every whole number is a multiple of unity:

For unity measures every whole number.

IX. Numbers equally augmented or diminished, continue to have the same difference.

Let any numbers, such as 6 and 8, be equally augmented, by the addition of 4 to each of them, the sums 10 and 12 will have the same difference as 6 and 8, viz. 2. Again, if from each of them 4 be taken away, the remainders 2 and 4 will have the same difference, viz. 2.

X. The difference of two unequal numbers added to the lesser, gives a sum equal to the greater; or subtracted from the greater, leaves a remainder equal to the lesser.

Let 3, the difference of two unequal numbers, 5 and 8, be added to 5, the lesser, and the sum will be 8, equal to the greater; or, if the difference 3 be subtracted from the greater, 8, the remainder will be 5, equal to the lesser.

XI. None

XI. None but similar or like things can be added or subtracted.

That is, units only can be added to units, tens only to tens, hundreds only to hundreds, &c.; for if unlike things were added together, the sum would be false. Thus, if 4 units were added to 5 tens, the sum would be 9; but neither 9 units, nor 9 tens, nor 9 of any other name.

In like manner, farthings only can be added to farthings, pence only to pence, shillings only to shillings, pounds only to pounds, ounces only to ounces, &c.; for if 3 farthings were added to 4 pence, the sum would be 7, but neither 7 farthings, nor 7 pence.

For the same reason, sheep can only be added to sheep, cows only to cows, horses only to horses, &c.; for if 7 sheep were added to 8 cows, the sum would be 15; but neither 15 sheep, nor 15 cows.

The same way of reasoning may be used with respect to subtraction.

XII. Any whole is equal to all its parts.

COROLLARIES.

1. If a significant figure, by itself, or with a cipher or ciphers annexed, be divided by 9, the remainder will be equal to the significant figure taken in its simple value.

Thus, if 8, 80, 800, 8000, &c. be divided by 9, the remainder will be 8: for if 8 be divided by 9; the quotient will be 0, and the remainder 8. Again, 80 is equal to 8 10's; that is, to 8 9's, and 8 1's, or 8. And, in like manner, 800 is equal to 80 10's; that is, to 80 9's, and 80 1's, or 80, whose remainder is already shown to be 8. The same way of reasoning may be applied to any other such number.

2. If any number be divided by 9, the remainder will be equal to the sum of the figures of the said number taken in their simple value, or to the excess above the 9's contained in the said sum.

For let the number be resolved by axiom VI. into its constituent

constituent parts, then, by the preceding corollary, the significant figure of each part will be the remainder of that part when divided by 9, and so the remainders of the several parts will be the figures of the given number; out of which, if need be, let the 9's be taken away, and the excess will be the remainder.

3. Hence the remainder arising from a number divided by 9 is found by adding the figures of the said number. Thus, the remainder of the number 231 divided by 9 is 6; for 2, 3, and 1, added together, make 6. Again, the remainder of 84632 is 5; for 8, 4, 6, 3, and 2, added together, make 23, whose excess above the 9's contained in it is 5, found by adding 2 to 3.

4. Numbers expressed by the same figures, however different the value of the numbers be, have the same remainder when divided by 9. Thus, 436, 463, 346, 364, 643, and 634, have all the same remainder when divided by 9, because the sum of their figures is the same.

VI. *An explanation of characters sometimes used for the sake of brevity.*

= Denoting equal to, is the sign or mark of an equation; and signifies, that the numbers betwixt which it stands are equal to one another.

+ Denoting plus, more, or added to, is the sign or mark of addition; and signifies, that the numbers it is placed between, are to be added together; as $6+2=8$; that is, 6 plus 2, or 6 having 2 added to it, is equal to 8.

— Denoting minus, or less, is the sign of subtraction; and shews, that the latter of the numbers betwixt which it stands is to be subtracted from the former; as $7-3=4$; that is, 7 minus 3, or 7 having 3 subtracted from it, is equal to 4.

× Denoting into, or multiplied into, is the sign of multiplication, and signifies, that the numbers betwixt which it stands are to be multiplied together; as $3\times 5=15$; that is, 3 into 5, or 3 multiplied into 5, is equal to 15.

C

÷ De-

$\div$ Denoting divided by, is the mark of division; and signifies, that the former of the numbers betwixt which it stands is to be divided by the latter; as $12 \div 4 = 3$; that is, 12 divided by 4, is equal to 3. Or this mark may be considered as setting the dividend above and the divisor below the line; and so it becomes a fractional expression, signifying, that the numerator is to be divided by the denominator; as $\frac{12}{4} = 3$; that is, 12 divided by 4 is equal to 3.

Division is also marked thus, $4)12(3$.

$::$ Denoting so is, is the sign of geometric proportion; and is placed betwixt the second and third terms of four proportional numbers; thus, $4:8::12:24$; that is, as 4 is to 8, so is 12 to 24.

$\sqrt{}$ This mark is called the *radical sign*; and has a figure or index set over it, to denote such a root of the number before which it stands. Thus, $\sqrt{25} = 5$ signifies, that the square root of 25 is 5; and $\sqrt[3]{64} = 4$ denotes the cube root of 64 to be 4. When the square root is meant, the radical sign is usually left blank, or has no figure placed over it.

VII. *Practical Questions.*

Q. 1. Troy was taken and destroyed by the Greeks, two thousand eight hundred and twenty years after the creation, and one thousand one hundred and sixty-four years after the flood, and four hundred and thirty-six years before the building of Rome, and eleven hundred and eighty-four years before the birth of Christ, and two thousand nine hundred and forty-four years before the accession of King George III. to the throne of Britain: How are these dates expressed in letters?

2. The inhabitants of a certain country are twenty-five millions four thousand and seventy men, thirty-four millions seven hundred and five thousand eight hundred and two women, fifty millions and four hundred boys, fifty-six millions six thousand and eight girls: How are these numbers expressed in figures?

3. A

3. A farmer has in his granary 470900608 grains of wheat, 80769000240 of barley, 702000678000 of rye, and 27300845302912 of oats: How are these numbers read, or expressed in words?

C H A P. II.

A D D I T I O N.

Addition is the collecting of two or more numbers into one sum or total.

I. *Addition of Integers.*

R U L E S.

I. Set figures of like place under other, *viz.* units under units, tens under tens, &c. See axiom XI.

II. Beginning at the lowest place, set down the right hand figure of the sum of every column, and carry the rest as so many units to the next superior place. See axioms II. III. IV.

* E X A M P L E I.

Because similar or like things only can be added, I place the numbers as directed in Rule I. *viz.* units under units, tens under tens, &c. as in the margin.

Then, beginning at the lowest place, *viz.* that of units, I say, 4 units and 3 units make 7 units, which I set below in the place of units; then 3 tens and 5 tens make 8 tens, which I set below in the place of tens; then 2 hundreds and 4 hundreds make 6 hundreds, which I set below in the place of hundreds, and find the sum or total to be 687.

From the repetition of the former example on the margin, it is plain, that it is a matter of indifference which of the numbers be placed uppermost.

453
234
—
687

E X A M P L E II.

Having placed the numbers, units under units, &c.

As in the margin, I say, 2 and 1 make 3, and 3 make 6, and 4 make 10; which being just 1 ten, and nothing over, I set the right-hand figure 0 in the place of units; and because ten in any lower place makes but one in the next superior place, I carry my 1 ten, as directed in Rule II. saying, 1 ten, collected out of the units, and 6 tens make 7 tens, and 4 make 11, and 0 makes but still 11, and 7 make 18; here again I set down the right-hand figure 8, in the place of tens, and carry the remaining figure 1, being 1 hundred, to the next place, viz. that of hundreds; and having in like manner added up this column, the amount is 31; so I set down the right-hand figure 1 in the place of hundreds, and carry the remaining figure 3 to the next place or column; which being also added, amounts to 24; I set the right-hand figure 4 below, in its proper place, and the remaining figure 2, which belongs to the next place, I set on the left hand, there being no figure in the next place to which it can be carried. So the sum or total is 24180.

The reason of the operation will still further appear by taking the sum of each column separately, and then adding them into one total, as in the margin.

	5974
	9803
	7541
	862
	—
	16 units
	17. tens
	30. hundreds
	21... thousands
	—
	24180 total

In adding integers, some teach young beginners to dot at every ten, and setting down the excess, to carry 1 for every dot. Thus, in adding the example in the margin, I say, 4 and 8 make 12; where I dot, set down the excess 2, and carry 1 for the dot to the next column; saying, 1 carried 7 8. 8. and 6 make 7, and 8 is 15; where I again dot, set down the excess 5, and carry 1 to 1 7 5 2

the

the next column; saying, 1 carried and 9 is 10; where I dot, and set down the 7 as the excess. The 1 carried for the dot is placed on the left hand. This method is too low for any but a child.

M O R E E X A M P L E S.

Ex. 3.

78943745

60738937

84976858

93543945

48732790

85476437

48763728

7845963

94769

Ex. 4.

8467472

7893968

47637

83745

4897

548763

674376

9848937

53700480

II. *Addition of the parts of integers, such as shillings, pence, farthings, ounces, &c.*

R U L E S.

I. Place like parts under other; *viz.* farthings under farthings, pence under pence, &c. See axiom XI.

II. Begin at the lowest of the parts, and carry according to the value of an unit of the next superior denomination; *viz.* for every four in the sum of farthings carry 1 to the pence, and for every twelve in the pence carry 1 to the shillings, &c. The reason of this rule is plain from the tables of coin, weights, and measures.

III. If you carry at 20, 30, 40, 60, or any just number of tens, as in adding shillings, degrees, poles, minutes, seconds, &c. proceed with the column of units as in addition of integers, and from the sum of the column of tens carry 1 for every two, or 1 for every three, &c. according as 20 or two tens, thirty or three tens, &c. make an unit of the next superior denomination. The reason appears plain in the following operations.

I. M O-

I. M O N E Y.

T A B L E.

4 farthings	}	make	1 penny
12 pence			1 shilling
20 shillings			1 pound

Marked thus.

$$l. \quad s. \quad d. \quad f. \text{ or } q.$$

$$1 = 20 = 240 = 960$$

l. is put for *libra*, a pound; *d.* for *denarius*, a penny; and *q.* for *quadrans*, a fourth-part; but *f.* is now the more usual mark for farthings.

There are a few other denominations of money, but never used in keeping accounts, *viz.* a groat, in value 4 *d.* a tetter 6 *d.* a crown 5 *s.* half a crown 2 *s.* 6 *d.* a noble 6 *s.* 8 *d.* an angel 10 *s.* a mark 13 *s.* 4 *d.* a guinea 21 *s.* half a guinea 10 *s.* 6 *d.* and a quarter of a guinea 5 *s.* 3 *d.*

That the learner may proceed in addition of money with the greater ease, it will be proper he get the following table by heart.

M O N E Y - T A B L E.

<i>f.</i>	<i>d.</i>	<i>d.</i>	<i>s.</i>	<i>s.</i>	<i>l.</i>
4 =	1	12 =	1	20 =	1
8 =	2	24 =	2	40 =	2
12 =	3	36 =	3	60 =	3
16 =	4	48 =	4	80 =	4
20 =	5	60 =	5	100 =	5
24 =	6	72 =	6	120 =	6
28 =	7	84 =	7	140 =	7
32 =	8	96 =	8	160 =	8
36 =	9	108 =	9	180 =	9
40 =	10	120 =	10	200 =	10

EX.

E X A M P L E.

(10) (20) (12) (4)

Having, according to Rule I. placed like parts under other, *viz.* farthings under farthings, pence under pence, &c. and in each of these denominations, units under units, tens under tens, as in the margin, I begin with the lowest of the parts, *viz.* the farthings; and say, 2 farthings and 1 farthing make 3 farthings, and 2 make 5, and 3 make 8; which, by the money-table, is 2 pence, and nothing over; wherefore I place 0 below in the place of farthings, or rather leave that place blank, and carry my 2 pence to the place of pence, as directed in Rule II. saying, 2 pence, collected out of the farthings, and 9 make 11, and 8 make 19, and 1 (passing the 0) make 20; to this sum of my units I add my tens. Thus, 20 and 1 ten make 30, and 1 ten more make 40 pence; which, by the money-table, is 3 twelves, or 3 shillings, and 4 pence over; these 4 pence I set below in the place of pence, and carry my 3 shillings to the place of shillings. Thus, 3 shillings, collected out of the pence, and 1 shilling make 4, and 7 make 11, and 9 make 20, and 8 make 28; and because in shillings we carry at a just number of tens, *viz.* at 20, I set my right-hand figure 8 below in the place of units, as directed in Rule III. and carry my 2 tens to the place of tens. Thus, 2 tens collected out of the units, and 1 ten make 3 tens, and 1 make 4, and 1 make 5 tens, or 2 twenties, and 1 ten over; and because 2 tens, or 1 twenty, make an unit in the next place, *viz.* that of pounds, I set my 1 ten below in the place of tens, and carry my 2 twenty shillings, or 2 pounds, to the place of pounds; which, being integers, are added as taught in addition of integers.

It is usual to subjoin the farthings to the pence by way of fraction, as in the margin, where the former example is transcribed in this form for the learner's instruction; in which $\frac{1}{4}$ denotes one farthing, $\frac{1}{2}$ two farthings, and $\frac{3}{4}$ three farthings.

L.	s.	d.
74	18	11 $\frac{3}{4}$
96	9	10 $\frac{1}{2}$
58	17	8 $\frac{1}{4}$
63	11	9 $\frac{1}{2}$
293	18	4
The		

The method of adding money explained above is the most approved, and generally practised by men of business; but yet other methods are sometimes used; such as,

1. Some, in instructing young children, teach them to dot at the figure which makes an unit of the next superior denomination; and, after setting down the overplus, they carry 1 for every dot. Thus, in adding the example in the margin, (10) (20) (12) (4) I begin with the farthings, and say, L. s. d. f.
 1 and 3 make 4; where I dot, and, 47 17. 10. 2
 setting down the 2 as an overplus, 98 14. 9. 3.
 I carry 1 for the dot, and say, 1 45 13 8
 carried and 8 make 9, and 9 is ———
 eighteen; where I dot, and say, 192 6 4 2
 6 of overplus and 10 make 16; where I again dot, and, setting down the overplus 4, I carry 2 for the dots, and proceed, saying, 2 carried and 13 make 15, and 4 make 19, and 1 ten on the left make 29; where I dot, and go on, saying, 9 of overplus and 7 make 16, and 1 ten on the left make 26; where I again dot, and, setting down the overplus 6, I carry 2 for the dots to the pounds, which are integers, and may be added either by dotting at every ten, or by the usual method of adding integers. This method of dotting in the addition of money is childish, and ought not to be too far indulged.

2. Some follow the common method in adding the farthings and pence; but in adding the shillings proceed thus: 2 shillings (10) (20) (12) carried from the pence and 4 make L. s. d.
 6, and 6 make 12, and 8 make 20; 84 18 11 $\frac{1}{2}$
 and to this sum of the units I add the 63 16 10 $\frac{1}{4}$
 tens on the left hand, saying, 20 and 78 10 9 $\frac{3}{4}$
 1 ten is 30, and 1 is 40, and 1 is 50; 95 4
 which, by the money-table, is 2 l. ———
 10 s.; wherefore I set down the 10 s. 322 10 7 $\frac{1}{2}$
 and carry the 2 l. to the pounds, which are integers, and added accordingly.

3. In adding up large accounts, some dot at 60 in the

the pence, and for every dot carry 5 to the shillings; and in adding the shillings they dot likewise at 60, and for every dot carry 3 to the pounds. Thus, in adding the example in the margin, I begin with the farthings, which I add in the usual manner, and find they amount to 23; which, by the money-table, is $5\frac{3}{4}$ d.; wherefore I set down the $\frac{3}{4}$, and carry 5 to the pence, which I proceed to add, by saying, 5 carried and 8 is 13, and 10 is 23, and 11 is 34, and 10 is 44, and 11 is 55, and 10 is 65; where I dot, and go on, saying, the overplus 5 and 4 is 9, and 9 is 18, and 11 is 29, and 10 is 39, and 6 is 45; which, by the money-table, is 3 s. 9 d.; accordingly I set down the 9 d. and 3 s. added to 5 s. for the dot make 8 to be carried to the shillings; which I proceed to add, by saying, 8 carried and 10 is 18, and 11 is 29, and 6 is 35, and 1 ten on the left is 45, and 8 is 53, and 1 ten on the left is 63; where I dot, and go on, saying, 3 of overplus and 10 is 13, and 9 is 22, and 3 is 25, and 1 ten on the left is 35, and 5 is 40, and 1 ten is 50, and 4 is 54, and 1 ten is 64; here again I dot, and go on, saying, 4 of overplus and 7 is 11, and 1 ten is 21, and 9 is 30, and 1 ten is 40, which is just 2 l. and no overplus to be set down; and these 2 l. with 6 l. for the two dots, make 8 to be carried to the pounds, which are integers, and may be either added in the usual method, or by dotting at every 100.

4. In casting up large accounts or bills, some chuse to divide them into parcels, and then, by any of the methods taught above, they cast up each parcel separately, and afterwards add the sums of the several parcels into one total; as is done by the following example.

D

(10)

	(10)	(20)	(12)
L.	s.	d.	
45	19	6	$\frac{1}{2}$
48	17	10	$\frac{3}{4}$
83	14	11	$\frac{1}{4}$
72	15	9	$\frac{1}{2}$
84	13	4	
86	9	10	$\frac{1}{2}$
78		11	$\frac{3}{4}$
64	10		
56	18	10	$\frac{1}{2}$
79	16	11	$\frac{3}{4}$
80	11	10	$\frac{1}{2}$
85	10	8	$\frac{3}{4}$
868		9	$\frac{3}{4}$

(10)	(20)	(12)			
L.	s.	d.			
42	14	8			
36	15	$4\frac{1}{2}$			
38	17	$10\frac{3}{4}$	L.	s.	d.
			118	7	$11\frac{1}{4}$
32	18	$9\frac{1}{4}$			
48	16	$11\frac{1}{2}$			
50	19	10			
			132	15	$6\frac{3}{4}$
56	8	$6\frac{3}{4}$			
54	6				
57		$8\frac{1}{4}$			
52	14	7			
			220	9	10
Total			471	13	4

It remains now to be observed, with respect to the empty places, whether in shillings, pence, or farthings, or those in the left-hand columns of the shillings and pence, that, instead of filling them up with ciphers, as the common practice was some time ago, the general fashion now, in most cases, is, to leave them blank; and this remark holds, not only in addition of money, but in every other kind of addition; nay, extends to every part of arithmetic, whether in addition, subtraction, multiplication, or division.

M O R E E X A M P L E S.

Ex. 1.			Ex. 2		
(10)	(20)	(12)	(10)	(20)	(12)
L.	s.	d.	L.	s.	d.
789	17	$10\frac{1}{2}$	483	15	$11\frac{1}{4}$
476	14	$11\frac{3}{4}$	724	13	4
894	18	9	865	18	$7\frac{1}{2}$
438	13	$10\frac{1}{4}$	978	14	$8\frac{3}{4}$
643	12	8	685	19	6
784	15		538	17	$4\frac{1}{4}$
435	6	$10\frac{1}{2}$	945	9	8
947	8	$6\frac{3}{4}$	782	5	10

Ex. 3.

Ex. 3.			Ex. 4.		
(10)	(20)	(12)	(10)	(20)	(12)
L.	s.	d.	L.	s.	d.
5748	19	11 $\frac{3}{4}$	4786	17	10 $\frac{1}{4}$
4875	17	10 $\frac{1}{2}$	8490	14	9 $\frac{1}{4}$
9387	13	4	5786	13	4 $\frac{3}{4}$
6453	14	8	8327	15	8
5874	10	7 $\frac{1}{2}$	7500	9	7 $\frac{1}{2}$
9482	9	11 $\frac{1}{2}$	6742	8	6
6093		10 $\frac{3}{4}$	894		10 $\frac{3}{4}$
7800	17	6	672	18	
508	15	10 $\frac{1}{4}$	95	16	11 $\frac{1}{2}$
75			8	19	10 $\frac{3}{4}$

2. AVOIRDupois WEIGHT.

T A B L E.

16 drams	}	make	1 ounce.
16 ounces			1 pound.
28 pounds			1 quarter.
4 quarters			1 hundred.
20 hundreds			1 tun.

Marked thus.

T.	C.	Q.	lb.	oz.	dr.
1	= 20	= 80	= 2240	= 35840	= 573440
	1	= 4	= 112	= 1792	= 28672

By Avoirdupois weight are weighed, butter, cheese, rosin, wax, pitch, tar, tallow, soap, salt, hemp, flax, beef, brass, iron, steel, tin, copper, lead, allum, and all grocery wares.

Note, 19 $\frac{1}{2}$ C. of lead make a fodder.

In adding the following example, I begin with the ounces, and say, 15 and 10 make 25; which being above 16, I dot, and carry away the excess 9, saying, 9 of excess and 6 make 15, and 8 make 23; where I again dot, and carry away the excess 7, saying, 7 and 2 is 9, and 1 ten on the left is 19; where I dot, and proceed with the excess 3, saying, 3 and 4 is 7, and 1 ten on the left is 17; where I dot, and carry the excess 1, saying,

D 2

I

1 and 5 is 6, and 1 ten on the left is 16; where I again dot, and there being no excess, I have nothing to set down.

(10)	(20)	(4)	(28)	(16)
T.	C.	Q.	lb.	oz.
74	19	3	27.	15.
85	17	2	24.	14.
68	13	1	20	12.
52	18	3	19.	8.
50	10	2	18.	6
48	9	3	16	10.
97	5	1	3	15

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I now proceed to add the pounds; saying, 5 carried from the ounces, *viz.* one for every dot, and 3 make 8, and 6 make 14, and 1 ten on the left is 24, and 8 make 32; which being above 28, I dot, and go on, saying, 4 of excess and 1 ten on the left is 14, and 9 is 23, and 1 ten on the left is 33; where I again dot, and go on, saying, 5 of excess and 20 is 25, and 4 is 29; where I dot, and proceed, saying, 1 of excess and 2 tens on the left make 21, and 7 make 28; where I dot, and the 2 tens, or 20, on the left, I set below.

I should now proceed to add the quarters; saying, 4 carried from the pounds and 1 make 5, &c.; but as you carry here 1 for every four, the quarters are added exactly as the farthings in addition of money. In the hundreds you carry at 20; which, therefore, are added as shillings. The tuns are integers; and added accordingly.

The excess may sometimes be known without actual addition. Thus, in casting up the ounces of the above example, it is easy to perceive, that 15 ounces wants only 1 to complete the pound; accordingly I imagine 1 to be taken from 10; and, after dotting, I go on with the excess 9, saying, 9 and 6 make 15; and again, as 15 wants only 1, I imagine this 1 to be taken from 8, and

and so I dot, and proceed with the excess 7, saying, 7 and 2 make 9, &c.

The accounts may be kept clean by making the dots on a blotter, or piece of waste paper laid under your hand.

Men of business frequently add both the ounces and the pounds as integers; divide the sum by 16 or 28; and, setting down the remainder as the excess, carry the quotient.

Retailers of silk, worsted, thread, and such other small wares, have occasion only for pounds, ounces, and drams, and this they call *avoirdupois small weight*; and in retailing some of these, it is usual to divide the ounce into four quarters instead of sixteen drams.

The denomination on the left hand, whether tuns, hundreds, or pounds, is always integers, and added as such; which in the following examples is pointed out by (10) superinscribed, being the number at which you carry in integers. This remark may be extended to other sorts of addition; as will appear in what follows.

M O R E E X A M P L E S.

Ex. 1.

Ex. 2.

(10)	(20)	(4)	(28)	(10)	(4)	(28)	(16)
T.	C.	Q.	lb.	C.	Q.	lb.	oz.
34	14	3	15	25	1	18	12
36	18	1	14	42	3	14	14
32	16	2	12	45	2	24	15
30	12	1	10	36	1	18	10
28	15	3	23	64	3	22	13
25	19	2	20	62	1	20	11
26	13	3	24	48	3	25	9
22	10	1	18	52	1	19	8
38	15	3	12	54	2	8	6

Ex. 3.

Ex. 3.

(10)(16)(16)

lb. oz. dr.

3 10 6

4 14 11

5 13 14

6 12 10

7 10 8

8 15 14

4 8 12

2 6 10

5 7 9

Ex. 4.

(10)(16)(4)

lb. oz. qrs.

4 12 3

7 10 2

3 14 1

5 13 3

6 11 2

8 10 3

4 13 2

8 15 3

3 8 1

3. TROY WEIGHT.

TABLE.

24 grains	}	make	{	1 penny-weight
20 penny-weights				1 ounce
12 ounces				1 pound Troy

Marked thus.

$$\begin{array}{cccc} \text{lb.} & \text{oz.} & \text{dw.} & \text{gr.} \\ 1 & = & 12 & = & 240 & = & 5760 \end{array}$$

By Troy weight are weighed gold, silver, jewels, amber, electuaries, and liquors.

Note. That 1 lb. Avoirdupois is equal to 14 oz. 11 dw. 15½ gr. Troy.

In adding the example in the margin, I begin with the grains, and say, 13 and 4 make 17, and 1 ten on the left make 27, which being above 24, I dot, and go on with the excess 3, saying 3 and 2 make 5, and 2 tens, or 20, on the left, make 25; where I again dot, and proceed with the excess 1, saying, 1 and 20 make 21, and 8 make 29; where I dot, and go on with the excess 5, saying, 5 and 1 ten on the left make 15, and 1 makes 16, and 20 on the left make

(10)	(12)	(20)	(24)
lb.	oz.	dw.	gr.
48	11	18	21.
42	10	14	18.
40	9	16	20
36	8	15	22.
38	10	10	14.
53	6	17	13

261 10 14 12

36. I set down the excess 12, and carry away 4 for the dots, to the penny-weights.

In casting up the penny-weights you carry at 20, which therefore are added exactly like shillings. In the ounces you carry at 12, which are therefore added as pence. The pounds are integers, and added as such.

Some men of business add the grains as integers, and, setting down the sum on a separate paper, divide it by 24, and then place the remainder as the excess under the grains, and carry the quotient to the penny-weights.

M O R E E X A M P L E S.

Ex. 1.

(10)	(12)	(20)	(24)
<i>lb.</i>	<i>oz.</i>	<i>dw.</i>	<i>gr.</i>
72	11	19	23
74	10	13	21
70	9	16	18
54	8	12	22
56	6	10	20
63	10	8	16
62	11	6	10
60	10	18	14

Ex. 2.

(10)	(12)	(20)	(24)
<i>lb.</i>	<i>oz.</i>	<i>dw.</i>	<i>gr.</i>
42	10	14	22
44	9	16	18
40	11	19	21
36	8	15	17
32	6	8	12
35	10	6	13
38	11	17	8
40	11	18	23

4. A P O T H E C A R I E S W E I G H T.

T A B L E.

20 grains	}	make	{	1 scruple
3 scruples				1 dram
8 drams				1 ounce
12 ounces				1 pound

Marked thus.

$$\text{lb} \quad \text{℥} \quad \text{ʒ} \quad \text{℥} \quad \text{gr.}$$

$$1 = 12 = 96 = 288 = 5760$$

By this weight apothecaries compound their medicines, their pound being the same as the pound Troy, but differently divided; but yet they buy and sell their drugs by Avoirdupois.

In

In adding the example in the margin, I begin with the grains; and because in grains we carry at 20, they are added as shillings; you add the scruples as integers, and carry 1 for every 3 of that sum; you add the drams in the same manner, and carry 1 for every 8; in the ounces you carry at 12, which are therefore added as pence; the pounds are integers, and added as such.

(10)	(12)	(8)	(3)	(20)
lb	3	3	9	gr.
44	11	7	2	19
43	10	5	1	15
48	9	3	2	14
36	8	2	1	18
32	10	6	2	12
30	11	4	1	17
238	2	7	1	15

M O R E E X A M P L E S.

Ex. 1.

(10)	(12)	(8)	(3)	(20)
lb	3	3	9	gr.
36	10	6	2	18
32	11	7	1	14
18	10	5	2	12
14	11	3	1	19
23	9	4	1	18
28	8	7	2	17
45	7	3	1	14
53	6	2		18

Ex. 2.

(10)	(3)	(20)
3	9	gr.
14	1	18
28	2	14
32	1	16
41	2	19
35	1	12
26	2	15
25	1	17
53	2	13

5. W O O L W E I G H T.

7 pounds	}	make	1 clove.
2 cloves			1 stone.
2 stones			1 todd.
6 $\frac{1}{2}$ todods			1 wey.
2 weys			1 sack.
12 sacks			1 laft.

Marked thus.

Laft. sack. wey. todd. stone. clove. lb.
 1 = 12 = 24 = 156 = 312 = 624 = 4368

Ex. 1.

Ex. 1.

(10)	(12)	(2)	(6 $\frac{1}{2}$)	(2)
<i>Last.</i>	<i>sack.</i>	<i>wey.</i>	<i>tod.</i>	<i>sto.</i>
72	11	1	6	1
70	10	1	3	1
74	9	1	5	1
63	8	1	3	1
82	11	1	5	1
86	10		4	1

Ex. 2.

(10)	()	(7)
<i>Ston.</i>	<i>clow.</i>	<i>lb.</i>
14	1	6
28	1	5
23	1	4
18	1	3
17	1	2
15	1	5

In casting up the todods, I say, 3 carried from the stones, and 4, is 7, and 5 is 12, and 3 is 15, and 5 is 20, and 3 is 23, and 6 is 29; in which I find 6 $\frac{1}{2}$ four times, making 26, and 3 of excess: accordingly I set down the excess 3, and carry 4 to the weys. The manner of adding the other denominations is obvious from the table, or from the superinscribed numbers.

Note. If the sum of the todods contain an odd number of weys, as 1, 3, 5, 7, &c. then the excess will be either $\frac{1}{2}$, 1 $\frac{1}{2}$, 2 $\frac{1}{2}$, 3 $\frac{1}{2}$, 4 $\frac{1}{2}$, or 5 $\frac{1}{2}$.

6. DRY MEASURE.

T A B L E.

2 pints	}	make	1 quart.
2 quarts			1 pottle.
2 pottles, or 8 pints			1 gallon.
2 gallons			1 peck.
4 pecks, or 8 gallons			1 bushel.
4 bushels			1 coomb
2 coombs, or 8 bushels			1 quarter.
5 quarters, or 40 bushels	}	make	1 load.

Marked thus.

Lo. Qrs. coom. bush. peck. gall. pot. qrts. pts.
 1 = 5 = 10 = 40 = 160 = 320 = 640 = 1280 = 2560

By this is measured grain; as wheat, barley, pease, &c.; also salt, sand, fruit, oysters, &c.

Note. That 33 $\frac{3}{4}$ cubic inches make a corn-pint,
 E 268 $\frac{4}{5}$

268 $\frac{4}{5}$ cubic inches make a corn-gallon, and 2150 $\frac{2}{3}$ cubic inches a Winchester bushel, which is a cylindrical vessel 18 $\frac{1}{2}$ inches wide, and 8 inches deep.

Ex. 1.

(10)	(5)	(8)	(4)
Load.	qrs.	bu.	pec.
28	4	7	3
18	3	6	2
14	2	5	1
15	3	4	3
42	4	7	2
48	2	6	1
36	3	5	3

Ex. 2.

(10)	(40)	(8)
Load.	bu.	gall.
34	39	7
36	32	6
24	28	5
18	36	4
25	38	3
20	35	5
32	34	6

In casting up the bushels in Ex. 2. because we carry at 40, a just number of tens, I add up the right-hand column as integers, set down the right-hand figure of the sum, and carry the rest to the left-hand column; which I also add up as integers, and carry 1 for every four of the sum.

In measuring coals the following table takes place.

4 pecks	} make {	1 bushel.
9 bushels		1 quarter.
4 quarters		1 chaldron.

Ex. 1.

(10)	(4)	(9)
Chal.	qrs.	bu.
54	3	8
28	2	7
52	1	6
46	3	5
48	2	7
32	3	8
36	2	5

Ex. 2.

(10)	(36)	(4)
Chal.	bu.	pec.
36	35	3
32	30	2
28	34	3
25	28	2
26	32	3
24	25	1
18	16	2

In casting up the bushels in Ex. 2. where we carry at 36,

36, I say, 4 carried from the pecks and 16 make 20, and 25 make 45; which being above 36, I dot, and carry the excess 9, saying, 9 and 32 is 41; where I again dot, and go on with the excess 5, saying, 5 and 28 make 33, and 4 make 37; where I dot, and proceed with the excess 1, saying, 1 and 3 tens, or 30, on the left, make 31, and as 31 wants only 5 to complete the chaldron, I imagine that 5 taken from 30; where I dot again, and proceed with the excess 25; and as the next number 35 wants only 1, I suppose that 1 taken from 25, and dotting again, I set down the excess 24, and carry 5 for the dots to the chaldrons, which are integers.

The man of business may either follow the above method; or he may add up the bushels as integers, divide their sum by 36, set down the remainder as the excess, and carry the quotient to the chaldrons.

The dry measures in most parts of *Scotland* differ from those used in *England*, and are divided as in the following table.

4 lippies or forpats	}	make	{	1 peck.
4 pecks				1 firlo.
4 firlots				1 boll.
16 bolls				1 chalder.

Ex. 1.

(10) <i>Ch.</i>	(16) <i>b.</i>	(4) <i>f.</i>	(4) <i>p.</i>
25	15	3	2
18	14	2	1
35	12	3	2
28	10	1	3
30	11	3	1
42	13	1	3
40	10	2	1

Ex. 2.

(10) <i>B.</i>	(4) <i>f.</i>	(4) <i>p.</i>	(4) <i>l.</i>
28	2	3	2
34	3	1	3
30	1	2	1
26	3	1	2
28	1	3	3
45	2	1	1
33	3	2	2

The bolls in Ex. 1. are added as ounces or drams in Avoirdupois weight; but the bolls in Ex. 2. are integers, and added as such.

268 $\frac{4}{5}$ cubic inches make a corn-gallon, and 2150 $\frac{2}{5}$ cubic inches a Winchester bushel, which is a cylindrical vessel 18 $\frac{1}{2}$ inches wide, and 8 inches deep.

Ex. 1.

(10)	(5)	(8)	(4)
Load.	qrs.	bu.	pec.
28	4	7	3
18	3	6	2
14	2	5	1
15	3	4	3
42	4	7	2
48	2	6	1
36	3	5	3

Ex. 2.

(10)	(40)	(8)
Load.	bu.	gall.
34	39	7
36	32	6
24	28	5
18	36	4
25	38	3
20	35	5
32	34	6

In casting up the bushels in Ex. 2. because we carry at 40, a just number of tens, I add up the right-hand column as integers, set down the right-hand figure of the sum, and carry the rest to the left-hand column; which I also add up as integers, and carry 1 for every four of the sum.

In measuring coals the following table takes place.

4 pecks	} make	1 bushel.
9 bushels		1 quarter.
4 quarters		1 chaldron.

Ex. 1.

(10)	(4)	(9)
Chal.	qrs.	bu.
54	3	8
28	2	7
52	1	6
46	3	5
48	2	7
32	3	8
36	2	5

Ex. 2.

(10)	(36)	(4)
Chal.	bu.	pec.
36	35	3
32	30	2
28	34	3
25	28	2
26	32	3
24	25	1
18	16	2

In casting up the bushels in Ex. 2. where we carry at 36,

36, I say, 4 carried from the pecks and 16 make 20, and 25 make 45; which being above 36, I dot, and carry the excess 9, saying, 9 and 32 is 41; where I again dot, and go on with the excess 5, saying, 5 and 28 make 33, and 4 make 37; where I dot, and proceed with the excess 1, saying, 1 and 3 tens, or 30, on the left, make 31, and as 31 wants only 5 to complete the chaldron, I imagine that 5 taken from 30; where I dot again, and proceed with the excess 25; and as the next number 35 wants only 1, I suppose that 1 taken from 25, and dotting again, I set down the excess 24, and carry 5 for the dots to the chaldrons, which are integers.

The man of business may either follow the above method; or he may add up the bushels as integers, divide their sum by 36, set down the remainder as the excess, and carry the quotient to the chaldrons.

The dry measures in most parts of *Scotland* differ from those used in *England*, and are divided as in the following table.

4 lippies or forpats	}	make	{	1 peck.
4 pecks				1 firlo.
4 firlots				1 boll.
16 bolls				1 chalder.

Ex. 1.

(10)	(16)	(4)	(4)
Gh.	b.	f.	p.
25	15	3	2
18	14	2	1
35	12	3	2
28	10	1	3
30	11	3	1
42	13	1	3
40	10	2	1

Ex. 2.

(10)	(4)	(4)	(4)
B.	f.	p.	l.
28	2	3	2
34	3	1	3
30	1	2	1
26	3	1	2
28	1	3	3
45	2	1	1
33	3	2	2

The bolls in Ex. 1. are added as ounces or drams in Avoirdupois weight; but the bolls in Ex. 2. are integers, and added as such.

7. WINE-MEASURE. T A B L E.

2	pints	}	make	{	1 quart.
4	quarts				1 gallon.
10	gallons				1 anchor.
18	gallons				1 runlet.
31 $\frac{1}{2}$	gallons				1 barrel.
42	gallons				1 tierce.
63	gallons				1 hoghead.
2	hogheads	}		{	1 pipe or butt.
2	pipes				1 tun.

Marked thus.

<i>Tun.</i>	<i>pipe.</i>	<i>hhd.</i>	<i>gall.</i>	<i>qrt.</i>	<i>pt.</i>
1	= 2	= 4	= 252	= 1008	= 2016

A puncheon, strictly speaking, is 4 anchors, or 80 gallons; but any cask betwixt a hoghead and a pipe is called a puncheon.

All spirits, mead, perry, cyder, vinegar, oil, and honey, are measured as wine.

The wine-pint contains $28\frac{7}{8}$ cubical inches, and the wine-gallon contains 231 cubical inches.

<i>Ex. 1.</i>				<i>Ex. 2.</i>		
(10)	(2)	(2)	(63)	(10)	(63)	(8)
<i>Tuns.</i>	<i>pip.</i>	<i>hhd.</i>	<i>gall.</i>	<i>Hhd.</i>	<i>gall.</i>	<i>pts.</i>
48	1	1	36	45	23	7
64	1	1	48.	52	38	6
52	1	1	30.	43	46	5
43	1	1	23	25	18	7
32	1	1	56.	73	25	6
45	1	1	38	65	30	7

In casting up the gallons, where we carry at 63, I proceed as follows, *viz.* in Ex. 1. I readily perceive that 46 wants only 7 to complete the hoghead; accordingly I imagine 7 taken from 38, and, after dotting at 56, I go on with the excess 31, saying, 31 and 3 make

34, and 2 tens, or 20, on the left, make 54; and as 54 wants only 9, I imagine 9 taken from 30, so I dot at 30, and proceed with the excess 21, saying, 21 and 8 make 29, and 40 on the left make 69; which being above 63, I dot again, and go on with the excess 6, saying, 6 and 36 make 42; which I set down, and carry 3 for the dots to the hogheads. The manner of operation is the same in Ex. 2.

The man of business may either cast up the gallons as directed above; or he may add them as integers, divide the sum by 63, set down the remainder as the excess, and carry the quotient.

8. BEER and ALE MEASURE.

T A B L E.

2	pints	}	make	{	1	quart.
4	quarts				1	gallon.
9	gallons				1	firkin.
2	firkins				1	kilderkin.
2	kilderkins				1	barrel.
1 $\frac{1}{2}$	barrels				1	hoghead.
2	hogheads				1	butt.
2	butts				1	tun.

Marked thus.

Tun. butt. hhd. barr. kild. firkin. gall. quart. Pt.
 1 = 2 = 4 = 6 = 12 = 24 = 48 = 96 = 192 = 384 = 768 = 1536 = 3072 = 6144 = 12288 = 24576 = 49152 = 98304 = 196608 = 393216 = 786432 = 1572864 = 3145728 = 6291456 = 12582912 = 25165824 = 50331648 = 100663296 = 201326592 = 402653184 = 805306368 = 1610612736 = 3221225472 = 6442450944 = 12884901888 = 25769803776 = 51539607552 = 103079215104 = 206158430208 = 412316860416 = 824633720832 = 1649267441664 = 3298534883328 = 6597069766656 = 13194139533312 = 26388279066624 = 52776558133248 = 105553116266496 = 211106232532992 = 422212465065984 = 844424930131968 = 1688849860263936 = 3377699720527872 = 6755399441055744 = 13510798882111488 = 27021597764222976 = 54043195528445952 = 108086391056891904 = 216172782113783808 = 432345564227567616 = 864691128455135232 = 1729382256910270464 = 3458764513820540928 = 6917529027641081856 = 13835058055282163712 = 27670116110564327424 = 55340232221128654848 = 110680464442257309696 = 221360928884514619392 = 442721857769029238784 = 885443715538058477568 = 1770887431076116955136 = 3541774862152233910272 = 7083549724304467820544 = 14167099448608935641088 = 28334198897217871282176 = 56668397794435742564352 = 113336795588871485128704 = 226673591177742970257408 = 453347182355485940514816 = 906694364710971881029632 = 1813388729421943762059264 = 3626777458843887524118528 = 7253554917687775048237056 = 14507109835375550096474112 = 29014219670751100192948224 = 58028439341502200385896448 = 116056878683004400771792896 = 232113757366008801543585792 = 464227514732017603087171584 = 928455029464035206174343168 = 1856910058928070412348686336 = 3713820117856140824697372672 = 7427640235712281649394745344 = 14855280471424563298789490688 = 29710560942849126597578981376 = 59421121885698253195157962752 = 118842243771396506390315925504 = 237684487542793012780631851008 = 475368975085586025561263702016 = 950737950171172051122527404032 = 1901475900342344102245054808064 = 3802951800684688204490109616128 = 7605903601369376408980219232256 = 15211807202738752817960438464512 = 30423614405477505635920876929024 = 60847228810955011271841753858048 = 121694457621910022543683507716096 = 243388915243820045087367015432192 = 486777830487640090174734030864384 = 973555660975280180349468061728768 = 1947111321950560360698936123457536 = 3894222643901120721397872246915072 = 7788445287802241442795744493830144 = 15576890575604482885591488987660288 = 31153781151208965771182977975320576 = 62307562302417931542365955950641152 = 124615124604835863084731911901282304 = 249230249209671726169463823802564608 = 498460498419343452338927647605129216 = 996920996838686904677855295210258432 = 1993841993677373809355710590420516864 = 3987683987354747618711421180841033728 = 7975367974709495237422842361682067456 = 15950735949418990474845684723364134912 = 31901471898837980949691369446728269824 = 63802943797675961899382738893456539648 = 127605887595351923798765477786913079296 = 255211775190703847597530955573826158592 = 510423550381407695195061911147652317184 = 1020847100762815390390123822295304634368 = 2041694201525630780780247644590609268736 = 4083388403051261561560495289181218537472 = 8166776806102523123120990578362437074944 = 16333553612205046246241981156724874149888 = 32667107224410092492483962313449748299776 = 65334214448820184984967924626899496599552 = 130668428897640369969935849253798993199104 = 261336857795280739939871698507597986398208 = 522673715590561479879743397015195972796416 = 1045347431181122959759486794030391945592832 = 2090694862362245919518973588060783891185664 = 4181389724724491839037947176121567782371328 = 8362779449448983678075894352243135564742656 = 16725558898897967356151788704486271129485312 = 33451117797795934712303577408972542258970624 = 66902235595591869424607154817945084517941248 = 133804471191183738849214309635890169035882496 = 267608942382367477698428619271780338071764992 = 535217884764734955396857238543560676143529984 = 1070435769529469910793714477087121352287059968 = 2140871539058939821587428954174242704574119936 = 4281743078117879643174857908348485409148239872 = 8563486156235759286349715816696970818296479744 = 17126972312471518572699431633393941636592959488 = 34253944624943037145398863266787883273185918976 = 68507889249886074290797726533575766546371837952 = 137015778499772148581595453067151533092743675904 = 274031556999544297163190906134303066185487351808 = 548063113999088594326381812268606132370974703616 = 1096126227998177188652763624537212264741949407232 = 2192252455996354377305527249074424529483898814464 = 4384504911992708754611054498148849058967797628928 = 8769009823985417509222108996297698117935595257856 = 17538019647970835038444217992595396235871190515712 = 35076039295941670076888435985190792471742381031424 = 70152078591883340153776871970381584943484762062848 = 140304157183766680307553743940763169886969524125696 = 280608314367533360615107487881526339773939048251392 = 561216628735066721230214975763052679547878096502784 = 1122433257470133442460429951526105359095756193005568 = 2244866514940266884920859903052210718191512386011136 = 4489733029880533769841719806104421436383024772022272 = 8979466059761067539683439612208842872766049544044544 = 17958932119522135079366879224417685745532099088089088 = 35917864239044270158733758448835371491064198176178176 = 71835728478088540317467516897670742982128396352356352 = 143671456956177080634935033795341485964256792704712704 = 287342913912354161269870067590682971928513585409425408 = 574685827824708322539740135181365943857027170818850816 = 1149371655649416645079480270362731887714054341637701632 = 2298743311298833290158960540725463775428108683275403264 = 4597486622597666580317921081450927550856217366550806528 = 9194973245195333160635842162901855101712434733101613056 = 18389946490390666321271684325803710203424869466203226112 = 36779892980781332642543368651607420406849738932406452224 = 73559785961562665285086737303214840813699477864812904448 = 147119571923125330570173474606429681627398955729625808896 = 294239143846250661140346949212859363254797911459251617792 = 588478287692501322280693898425718726509595822918503235584 = 1176956575385002644561387796851437453019191645837006471168 = 2353913150770005289122775593702874906038383291674012942336 = 4707826301540010578245551187405749812076766583348025884672 = 9415652603080021156491102374811499624153533166696051769344 = 18831305206160042312982204749622999248307066333392103538688 = 37662610412320084625964409499245998496614132666784207077376 = 75325220824640169251928818998491996993228265333568414154752 = 150650441649280338503857637996983993986456530667136828309504 = 301300883298560677007715275993967987972913061334273656619008 = 602601766597121354015430551987935975945826122668547313238016 = 1205203533194242708030861103975871951891652245337094626476032 = 2410407066388485416061722207951743903783304490674189252952064 = 4820814132776970832123444415903487807566608981348378505904128 = 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315936875005671560454042053240650976956685286201647333762932932608 = 631873750011343120908084106481301953913370572403294667525865865216 = 1263747500022686241816168212962603907826741144806589335051731730432 = 2527495000045372483632336425925207815653482289613178670103463460864 = 5054990000090744967264672851850415631306964579226357340206926921728 = 10109980000181489934529345703700831262613929158452714680413853843456 = 20219960000362979869058691407401662525227858316905429360827707686912 = 40439920000725959738117382814803325050455716633810858721655415373824 = 80879840001451919476234765629606650100911433267621717443310830747648 = 161759680002903838952469531259213300201822866535243434886621661495296 = 323519360005807677904939062518426600403645733070486869773243322990592 = 647038720011615355809878125036853200807291466140973739546486645981184 = 1294077440023230711619756250073706401614582932281947479092973291962368 = 2588154880046461423239512500147412803229165864563894958185946583924736 = 5176309760092922846479025000294825606458331729127789916371893167849472 = 10352619520185845692958050000593651212916663458255579832743786335698944 = 20705239040371691385916100001187302425833326916511159665487572671397888 = 41410478080743382771832200002374604851666653833022319330975145342795776 = 82820956161486765543664400004749209703333307666044638661950290685591552 = 165641912322973531087328800009498419406666615332089277323900581371183104 = 331283824645947062174657600018996838813333230664178554647801162742366208 = 662567649291894124349315200037993677626666461328357109295602325484732416 = 1325135298583788248698630400075987355253332922656714218591204650969464832 = 2650270597167576497397260800151974710506665845313428437182409301938929664 = 5300541194335152994794521600303949421013331690626856874364818603877859328 = 10601082388670305989589043200607898842026663381253713748729637207755718656 = 21202164777340611979178086401215797684053326762507427497459274415511437312 = 42404329554681223958356172802431595368106653525014854994918548831022874624 = 84808659109362447916712345604863190736213307050029709989837097662045749248 = 169617318218724895833424691209726381472426614100059419979674195324091498496 = 339234636437449791666849382419452762944853228200118839959348390648182996992 = 678469272874899583333698764838905525889706456400237679918696781296365993984 = 1356938545749799166667397529677811051779412912800475359837393562592731987968 = 2713877091499598333334795059355622103558825825600950719674787125185463975936 = 5427754182999196666669590118711244207117651651201901439349574250370927951872 = 10855508365998393333339180237422488414235303302403802878699148500741855903744 = 21711016731996786666678360474844976828470606604807605757398297001483711807488 = 43422033463993573333356720949689953656941213209615211514796594002967423614976 = 86844066927987146666713441899379907313882426419230423029593188005934847229952 = 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Ex. 1.				Ex. 2.		
(10)	(3)	(2)	(9)	(10)	(3)	(36)
<i>Hhd.</i>	<i>kild.</i>	<i>firk.</i>	<i>gal.</i>	<i>But.</i>	<i>bar.</i>	<i>gall.</i>
32	2	1	8	68	2	35
30	1	1	7	42	1	32
28	1	1	5	40	2	28
40	2	1	6	34	1	15
42	1	1	7	22	2	12
36	2	1	6	18	1	24
37	1	1	7	9	2	18

The gallons in Ex. 2. are added like the bushels in Ex. 2. of coal-measure.

In *Scotland* the denominations of liquid measure are as in the following

T A B L E.

4 gills	}	make	1 mutchkin.
2 mutchkins			1 chopin.
2 chopins			1 pint.
2 pints			1 quart.
4 quarts			1 gallon.
16 gallons	}	}	1 hoghead.

The *Scots* mutchkin is somewhat less than the *English* pint.

Ex. 1.				Ex. 2.			
(10)	(16)	(4)	(2)	(10)	(2)	(2)	(4)
<i>Hhd.</i>	<i>gall.</i>	<i>qrt.</i>	<i>pt.</i>	<i>Pt.</i>	<i>chop.</i>	<i>mut.</i>	<i>g.</i>
42	14	3	1	14	1	1	3
48	12	2	1	23	1	1	2
50	15	1	1	25	1	1	3
52	10	3	1	26	1	1	2
45	13	1	1	15	1	1	1
42	14	2	1	18	1	1	3
28	12	3	1	20	1	1	2
22	10	3	1	24	1	1	3

9. CLOTH-MEASURE.

T A B L E.

4 nails	}	make	1 quarter.
4 quarters			1 yard.
3 quarters			1 ell Flemish.
5 quarters			1 ell English.

A nail is equal to $2\frac{1}{4}$ inches, and consequently a quarter is equal to 9 inches.

Hollands are generally measured by the ell English; tapistry by the ell Flemish; and most other things by the yard.

Ex. 1.	Ex. 2.	Ex. 3.
(10) (4) (4)	(10) (3) (4)	(10) (5) (4)
<i>Yds. qrs. n.</i>	<i>El. Fl. qrs. n.</i>	<i>El. Eng. qrs. n.</i>
42 3 3	63 2 3	69 4 3
36 2 1	52 1 3	62 3 2
30 1 3	62 2 2	40 1 3
28 2 2	48 1 1	45 4 2
24 1 3	42 2 1	36 2 1
25 3 1	40 2 2	34 1 3

10. LONG MEASURE.

T A B L E.

3 barley-corns, or 12 lines	}	make	1 inch.
12 inches			1 foot.
3 feet			1 yard.
2 yards, or 6 feet,			1 fathom.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet,			1 pole, or perch.
4 poles, or 66 feet,			1 chain.
10 chains, or 40 poles			1 furlong.
8 furlongs, or 80 chains,			1 mile.
3 miles			1 league.

A hand, or hand's breadth, in horsemanship, is 4 inches.

A common pace is 2 feet 6 inches.

A geometrical pace is 5 feet.

The

The chain is divided into 100 equal parts, or links; each link being $7\frac{92}{100}$ inches.

Ex. 1.

(10)	(3)	(8)	(40)
Leag.	miles.	fur.	poles.
20	2	7	38
18	2	5	24
25	1	6	35
24	2	7	32
18	1	4	26
12	2	3	20
17	2	3	33

Ex. 2.

(10)	(5 $\frac{1}{2}$)	(3)	(12)
Poles.	yds.	feet.	inches.
36	4	2	11
28	3	1	10
25	1	2	11
16	2	1	9
24	3	1	7
36	2	2	8
23	4	2	5

In casting up the poles in Ex. 1. where we carry at 40, a just number of tens, you add as in integers; and from the sum of the left-hand column carry 1 for every four, setting down the excess. The poles in Ex. 2. are integers, and added as such.

The denominations of long measure used in *Scotland* are as in the following

T A B L E.

37 inches	} make {	1 ell.
6 ells, or $18\frac{1}{2}$ feet,		1 fall.
4 falls, or 74 feet,		1 chain.
10 chains, or 40 falls,		1 furlong.
8 furlongs, or 80 chains,		1 mile.

The chain is divided into 100 links; each link being $8\frac{88}{100}$ inches.

Ex. 1.

(10)	(8)	(10)	(4)
Miles.	fur.	ch.	falls.
24	7	9	3
22	5	8	2
18	4	6	3
14	6	5	3
28	3	7	2
23	2	8	3

Ex. 2.

(10)	(4)	(6)	(37)
Ch.	falls.	ells.	inches.
36	3	5	36.
32	2	4	32.
45	1	3	25.
23	2	4	18
18	3	5	16.
24	2	2	24

In

In casting up the chains in Ex. 1. because you carry at 10, you work as in addition of integers. In Ex. 2. the chains are integers.

In adding the inches in Ex. 2. I say, 24 and 6 make 30, and 1 ten on the left make 40; which, being above 37, I dot at 16, and go on with the excess 3, saying, 3 and 18 make 21, and 25 make 46; so I dot at 25, and proceed with the excess 9, saying, 9 and 32 make 41; accordingly I dot at 32, and go on with the excess 4, saying, 4 and 36 make 40; so I dot at 36, set down the excess 3, and carry 4 for the dots to the ells.

Men of business are at liberty to add the inches as above directed; or they may cast them up as integers, divide the sum by 37, and, setting down the remainder as the excess, carry the quotient.

11. LAND-MEASURE.

T A B L E.

30 $\frac{1}{4}$ square yards	} make {	1 square pole.
40 square poles		1 rood of land.
4 roods		1 acre.

Note, Customary measure in some places differs from the statute measure contained in the above table.

Land is usually measured by a chain of 100 links, whose length is 4 poles, or 66 feet; and so 10 square chains is equal to an acre.

Ex. 1.

(10)	(4)	(40)	(30 $\frac{1}{4}$)
Acres.	roods.	sq. p.	sq. y.
25	3	39	15
23	2	25	24.
48	1	32	18
64	3	35	14.
52	1	31	22.
43	2	36	26

Ex. 2.

(10)	(4)	(40)
Acres.	roods.	sq. p.
48	2	35
34	1	27
30	3	25
46	3	38
42	1	34
39	2	32

In adding the square yards in Ex. 1. I say, 26 and 22 make 48; which being above 30 $\frac{1}{4}$, I dot at 22, and

F

go

go on with the excess $17\frac{3}{4}$, saying, $17\frac{3}{4}$ and 14 make $31\frac{3}{4}$; so I dot again at 14, and proceed with the excess $1\frac{1}{2}$, saying, $1\frac{1}{2}$ and 18 make $19\frac{1}{2}$, and 24 make $4\frac{1}{2}$; so I dot at 24, and go on with the excess $13\frac{1}{4}$, saying, $13\frac{1}{4}$ and 15 make $28\frac{1}{4}$; which I set down as the excess, and carry 3 for the dots to the poles.

In casting up the square poles, whether in Ex. 1. or 2. you carry at 40, a just number of tens, which therefore are added as integers; and from the sum of the left-hand column you carry 1 for every four.

The denominations of land-measure used in *Scotland* are contained in the following

T A B L E.

36 square ells	} make	1 square fall.
40 square falls		1 rood of land.
4 roods		1 acre.

Note, That customary measure in some places differs from this table.

The *Scots* chain consists of 100 links, being in length 4 falls, or 74 feet; and consequently 10 square chains make an acre.

Four *Scots* acres are somewhat more than five *English* acres.

Ex. 1.			
(10)	(4)	(40)	(36)
Acr.	roo.	sq.f.	sq.el.
72	3	38	34
68	1	35	26
62	2	24	30
43	3	37	35
41	1	33	28
45	2	28	32

Ex. 2.		
(10)	(4)	(40)
Acr.	roo.	sq.f.
42	2	34
28	3	32
23	1	35
46	3	26
35	2	28
53	1	25

The square ells in Ex. 1. are added like the bushels in coal-measure.

12. SUPERFICIAL MEASURE.

T A B L E.

144 square inches } make { 1 square foot.
 9 square feet } 1 square yard.

By this measure are measured board, glass, pavements, plaistering, wainscoting, tiling, flooring, &c.

Ex. 1.		Ex. 2.		Ex. 3.	
(10)	(9)	(10)	(9)	(10)	(9)
Yar.	fee.	Yar.	fee.	Yar.	fee.
72	8	42	5	38	5
43	7	36	3	32	6
34	5	22	7	74	7
68	6	18	8	85	4
35	4	24	6	42	8
32	7	35	4	26	7

If there be inches, they must be added as integers, then dividing their sum by 144, the remainder will be the excess, and the quotient must be carried to the feet.

13. S O L I D M E A S U R E.

1728 solid inches } make { 1 solid foot.
 27 solid feet } 1 solid yard.

By this is measured timber, stone, digging, &c.

Ex. 1.		Ex. 2.		Ex. 3.	
(10)	(27)	(10)	(27)	(10)	(27)
Yar.	fee.	Yar.	fee.	Yar.	fee.
52	26.	43	25	67	23
48	22.	42	18	25	24
34	18.	34	14	32	18
25	23	26	23	27	23
68	12.	38	20	16	15
32	16	45	16	14	14

In adding the feet, I say, (Ex. 1.), 16 and 12 make 28;
 F 2 which

which being above 27, I dot at 12, and go on with the excess 1, saying, 1 and 23 make 24, and 8 make 32; where I dot again, and proceed with the excess 5, saying, 5 and 1 ten on the left, make 15, and as 22 wants only 5, I take that 5 from the 15, and, dotting at 22, I go on with the excess 10; and as 26 wants only 1, I take that 1 from 10; so I dot at 26, and, setting down the excess 9, I carry 4 for the dots to the yards.

If inches be given, they must be added as integers, and their sum being divided by 1728, the remainder will be the excess, and the quotient must be carried to the feet.

14. A C I R C L E.

T A B L E.

60 seconds	}	make	{	1 minute.
60 minutes				1 degree.
30 degrees				1 sign.
12 signs				1 circle.

Marked thus.

$$\text{Circ.} \quad \text{fig.} \quad ^\circ \quad ' \quad ''$$

$$1 = 12 = 360 = 21600 = 1296000$$

Ex. 1.

(10) (12) (30) (60)

Circ.	fig.	°	'
15	11	29	59
24	5	24	56
36	7	18	42
52	6	20	35
19	10	15	28
28	4	12	45

Ex. 2.

(10) (30) (60) (60)

Sig.	°	'	''
10	25	48	54
9	23	40	36
5	18	34	48
2	22	32	40
7	20	30	25
3	18	15	18

In casting up the seconds, minutes, and degrees, where you carry at 60 or 30, a just number of tens, you add as in integers, and from the sum of the left-hand column, you carry 1 for every 6 in the seconds and minutes, and 1 for every 3 in the degrees.

15. TIME.

15. T I M E.

T A B L E.

60 seconds	}	make	{	1 minute.
60 minutes				1 hour.
24 hours				1 day.
7 days				1 week.
4 weeks				1 month.
13 months				1 year of 364 days.

But the year complete consists of 365 days and 6 hours; and is divided into twelve unequal calendar months; whose names, with the number of days they contain, are as in the margin.

The 6 hours, in the space of 4 years, make a day; and is accordingly every fourth year added to February, which then consists of 29 days; and this year is called *Bissex-tile* or *Leap-year*, and contains 366 days.

Names.	Days.
January	31
February	28
March	31
April	30
May	31
June	30
July	31
August	31
September	30
October	31
November	30
December	31
	365

Ex. 1.

(10)	(13)	(4)	(7)
Year.	mon.	wee.	da.
25	11.	3	6
26	10.	2	4
32	12.	3	2
34	11.	1	5
38	9	2	6
42	7.	3	3
45	4	2	4

Ex. 2.

(10)	(24)	(60)	(60)
Da.	ho.	min.	sec.
38	23	56	48
34	20	45	54
32	18	42	36
25	12	25	18
18	16	15	45
20	21	38	42
15	14	50	25

In

In adding the months in Ex. 1. I say, 5 carried from the weeks, and 4 make 9, and 7 make 16; which being above 13, I dot at 7, and go on with the excess 3, saying, 3 and 9 make 12, and 1 make 13; where I dot, and proceed, saying, 1 ten on the left, and 12 make 22; where I again dot, and go on with the excess 9, saying, 9 and 10 make 19; where I dot, and proceed with the excess 6, saying, 6 and 11 make 17; where I dot, and setting down the excess 4, I carry 5 for the dots to the years.

In Ex. 2. the hours are added like the grains in Troy weight, and the minutes and seconds are added exactly like those of a circle.

The learner, by this time, may be supposed to be pretty well skilled in addition; and will henceforth, in order to his casting up any account, want only to know, how many of any inferior denomination makes an unit of the next superior; and this he will acquire partly by reading books, and partly by conversation and practice; and as a large detail of this kind would be tedious, and does not properly belong to arithmetic, I shall only add two or three things more.

12 things of any kind	}	make	{	1 dozen.
12 dozen				1 small gros.
12 small gros				1 great gros.

24 sheets of paper	}	make	{	1 quire.
20 quires				1 ream.

20 things of any sort	}	make	{	1 score.
5 score				100, or the short hundred.
6 score				120, or the long hundred.

III. *The Proof of Addition.*

Addition may be proved several ways.

1. Merchants and men of business usually add each column first upwards and then downwards, and upon finding

finding the sum to be the same both ways, they conclude the work to be right: and this is all the proof that their time, or the hurry of business, will admit of.

2. It is a common practice in schools, to prove the work by a second summing without the top-line; and if this sum added to the top-line make the first total, the work is supposed to be right; as in the two following examples.

Ex. 1.

8745	Top-line
------	----------

 9378

4764

5876

 28763

Total

 20018

Total without the top-line

 28763

Proof

Ex. 2.

L.	s.	d.
748	15	10 $\frac{3}{4}$

 674 13 11 $\frac{1}{2}$
835 17 9 $\frac{1}{4}$

90 18 8

 2350 6 3 $\frac{1}{2}$

 1601 10 4 $\frac{3}{4}$

 2350 6 3 $\frac{1}{2}$

3. Addition is also proved by casting out the 9's: for if the excess above the 9's in the total be the same as the excess in the items, the work may be presumed right. Thus, to prove the example in the margin, I begin with the items, and say, $3 + 4 = 7$, and $7 + 7 = 14 = 1 + 4 = 5$; with this 5 I pass to the next item, and say, $5 + 6 = 11 = 1 + 1 = 2$, and $2 + 8 = 10 = 1$, and $1 + 4 = 5$: which 5, being the excess of the items, I place at the top of the cross, and proceed to cast the 9's out of the total, saying, $1 + 3 = 4$, and $4 + 1 = 5$: which 5, being the excess of the total, I place at the foot of the cross; and because it is the same with the figure at the top, I conclude the work to be right.

If the items are of different denominations; as pounds, shillings, pence, &c.; you must begin with the highest denomination;

denomination; and, after casting out the 9's, reduce the excess to the next inferior denomination; and then casting out the 9's, reduce the excess to the next inferior denomination; proceed in like manner with this, and all the other lower denominations, placing the last excess at the top of the cross; then, in the same manner, cast the 9's out of the total, placing the excess at the foot of the cross; and if the figure at the foot and top be the same, the work may be presumed right.

Thus, to prove the example in the margin, I begin with the pounds, and say, $4 + 8$

$= 12 = 1 + 2 = 3$, and $3 + 5$ L. s. d. f.

$= 8$, and $8 + 5 = 13 = 1 +$ 48 17 10 1

$3 = 4$; and the excess 4 reduced to shillings is $4 \times 20 = 80$ 55 18 7 2

$= 8$, and $8 + 1 = 9 = 0$; then 104 16 5 3

$7 + 1 = 8$, and $8 + 8 = 16$

$= 1 + 6 = 7$; and the excess 7 reduced to pence is

$7 \times 12 = 84 = 8 + 4 = 12 = 1 + 2 = 3$, and

$3 + 1 = 4$, and $4 + 7 = 11 = 1 + 1 = 2$; and the

excess 2 reduced to farthings is $2 \times 4 = 8$, and $8 + 1$

$= 9 = 0$, and $0 + 2 = 2$: so this 2, being the excess

of the items, I place at the top of the cross, and proceed to cast the 9's out of the total, saying, $1 + 4 = 5$,

and the excess 5 reduced to shillings is $5 \times 20 = 100$

$= 1$, and $1 + 1 = 2$, and $2 + 6 = 8$; and this excess

8 reduced to pence is $8 \times 12 = 96 = 6$, and $6 + 5$

$= 11 = 1 + 1 = 2$; and this excess 2 reduced to far-

things is $2 \times 4 = 8$, and $8 + 3 = 11 = 1 + 1 = 2$:

which excess 2 I place at the foot of the cross; and be-

cause it is the same with the figure at the top, I conclude

the work to be right.

The method of proof by casting out the 9's is found-

ed on the corollaries deduced from the axioms; and if

any operation, whether in addition, subtraction, multi-

plication, or division, be right, this kind of proof will

always show it to be so; but if an operation be wrong,

by a figure or figures being misplaced, or by miscounting

9, or any just number of 9's, this kind of proof will not

discover the mistake.

IV. *Practical Questions.*

1. A person dying, left for the use of his widow L. 4000; to each of his four sons L. 4256; to each of his five daughters L. 3678; to each of six near relations L. 245; What was his estate? *Ans.* L. 40,884.

2. H of Hamburg is debtor to L of London: For broad cloth, L. 428 : 14 : 10; for kerseys, L. 273, 17. 6½.; for fustians, L. 364 : 19 : 8; for druggets, L. 568 : 18 : 4¾; for muslin, L. 392 : 12 : 8½; for grocery-wares, L. 683 : 1 : 8¼; the factorage came to L. 104, 6s.; custom, wharfage, and incident charges, L. 4, 15s. For what sum must L draw on H? *Ans.* L. 2821 : 19 : 10.

3. B buys six bags of hops, of which N° 1. weighed C. 2 : 3 : 14; N° 2. C. 2 : 1 : 24; N° 3. C. 2 : 2 : 20; N° 4. C. 2 : 1 : 22; N° 5. C. 2 : 0 : 26; N° 6. C. 2, 3q. 12 lb.; as also a couple of pockets ditto, that weighed 54½ lb. each. How many hundred weight did he purchase? *Ans.* C. 16 : 2 : 3.

4. A goldsmith sells five dozen of silver spoons, weighing 12 lb. 10 oz. 14 dw.; two tankards, weighing 1 lb. 8 oz. 18 dw.; ten salts, weighing 4 lb. 11 oz. 16 dw.; forty-two plates, weighing 38 lb. 6 oz. 10 dw.; a pair of jugs, weighing 2 lb. 4 oz. 13 dw.; two tea kettles, weighing 13 lb. 5 oz. 12 dw. What quantity did he sell? *Ans.* 74 lb. 3 dw.

5. A wine-merchant imports 12 tuns 2 hhds 45 gallons of claret; 14 tuns 3 hhds 48 gallons of Malaga; 16 tuns 1 hhd 54 gallons of Port; 20 tuns 2 hhds 56 gallons of Canary; 18 tuns 3 hhds of Madeira; and 10 tuns 2 hhds 42 gallons of sherry. What quantity did he import in all? *Ans.* 94 tuns. 56 gallons.

6. A certain farm consists of six inclosures, whereof the first contains 42 acres 3 roods 36 poles; the second, 36 a. 2 r. 24 p.; the third, 45 a. 1 r. 38 p.; the fourth, 52 a. 2 r. 28 p.; and each of the other two contains 26 a. 3 r. 14 p. How many acres in all? *Ans.* 231 a. 1 r. 34. p.

7. A gentleman had seven sons; from the birth of the

the first to the birth of the second there intervened 384 days 18 hours 50 minutes; from the birth of the second to that of the third, 582 d. 22 h. 58 m.; from the birth of the third to that of the fourth, 623 d. 12 h. 48 m.; from the birth of the fourth to that of the fifth, 592 d. 16 h. 36 m.; from the birth of the fifth to that of the sixth, 745 d. 19 h. 45 m.; from the birth of the sixth to that of the seventh, 864 d. 17 h. 54 m. What was the age of the eldest when the youngest was born? *Ans.* 3794 days, 12 hours, and 51 minutes.

C H A P. III.

SUBTRACTION.

SUBTRACTION is the taking a lesser number from a greater, in order to discover their difference, or the remainder.

I. *Subtraction of Integers.*

R U L E S.

I. Set figures of like place under other, *viz.* units under units, tens under tens, &c. and the greater of the given numbers uppermost. See axiom XI.

II. Beginning at the place of units, take the lower figures from those above, borrowing and paying ten, as need requires, and write the remainders below. See axioms II. III. IX.

E X A M P L E I.

Because similar or like	867	major or minuend.
things only can be sub-	562	minor or subtrahend.
tracted, I place the num-	<u>305</u>	
bers as directed in Rule I.		difference or remainder.

viz. units under units, tens under tens, &c. and the greater uppermost, as in the margin. Then, beginning at the place of units, I say, 2 units from 7 units, and 5 units remain; which I set below, in the place of units; then 6 tens from 6 tens, and nothing remains; wherefore I set 0 below, in the place of tens; then 5 hundred from

from 8 hundred, and 3 hundred remain; which I set below, in the place of hundreds; and find the total difference or remainder to be 305.

EXAMPLE II.

Having placed the numbers, units under units, &c. as in the margin, I say, 5 units from 2 units I cannot, but, because an unit in the next superior place makes ten in this place, I borrow 1, viz. 1 ten, from the said next place, as directed in Rule II.; which 1 ten being added to 2 makes 12; then I say, 5 from 12, and 7 remains; which 7 I set below, in the place of units: then I proceed, and pay the unit borrowed, either by esteeming 3, the next figure in the major, to be only 2, or, which is more usual, and the same in effect, by adding 1 to the next figure in the minor, thus, 1 that I borrowed and 8 make 9, from 3 I cannot, but, borrowing as before, I say, 9 from 13, and 4 remains; which 4 I set below: I proceed, and say, 1 that I borrowed and 7 make 8, from 4 I cannot, but from 14, and 6 remains; which 6 I set below: I go on, and say, 1 borrowed and 2 make 3, from 7, and 4 remains; which 4 I set below. So the difference or remainder is 4647. By borrowing and paying in this manner the major and minor are equally augmented, or have the same number added to each of them; and consequently continue to have the same difference, by axiom IX.

EXAMPLE III.

Some, instead of adding 10 to the upper lesser figure, subtract directly from 10, and add the difference to the upper figure for the remainder, thus, 3 from 2 I cannot, but 3 from 10, and 7 remains; which 7 added to 2 gives 9 for a remainder: then I go on, saying, 1 borrowed and 7 make 8, which from 5 I cannot, but from 10, and 2 remains; which 2 added to 5 gives 7 for a remainder: so I proceed, saying, 1 borrowed and 8 make 9, which from 4 I cannot, but from 10, and 1 remains; which

$$\begin{array}{r} 17452 \\ 873 \\ \hline 16579 \end{array}$$

1 added to 4 gives 5 for a remainder : so I go on, and say, 1 borrowed from 7, and 6 remains, and 0 from 1, and 1 remains. This method is the same in effect with the other ; but not so usual, and somewhat childish.

MORE EXAMPLES.

	<i>Ex. 4.</i>	<i>Ex. 5.</i>	<i>Ex. 6.</i>
From	84765	94307	742680
Take	20857	5768	8794

II. *Subtraction of the parts of integers ; such as shillings, pence, farthings, ounces, &c.*

RULES.

I. Place like parts under other, *viz.* farthings under farthings, pence under pence, &c. and the greater of the given numbers uppermost. See axiom XI.

II. Begin at the lowest of the parts, and borrow according to the value of an unit of the next superior denomination ; *viz.* in farthings borrow 4, in pence borrow 12, &c. as the tables of coin, weights, and measures direct.

III. If you borrow 20, 30, 40, 60, or any just number of tens, as in subtracting shillings, degrees, poles, minutes, seconds, &c. proceed with the right-hand column, as in subtraction of integers ; and then subtract your tens, borrowing, if need be, the number of tens contained in an unit of the next superior denomination. The reason appears plain in the following operations.

I. MONEY.

Having, according to Rule I. placed like parts under other, *viz.* farthings under farthings, pence under pence, &c. and in each of these denominations, units under units, tens under tens, and the greater of the

(10)	(20)	(12)	(4)	
<i>L.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>	
73	15	10	2	major.
48	12	6	2	minor.
25	3	4		rem.

the given numbers uppermost, as in the margin, I begin with the farthings, and say, 2 from 2, and 0 remains; so, having nothing to set down, I leave the place blank, and proceed to the pence, saying, 6 from 10 and 4 remains; which 4 I set down, and go on to the shillings, saying, 2 from 5, and 3 remains, and 1 from 1, and 0 remains; or I may say at once, 12 from 15, and 3 remains; which 3 being set down, I proceed to the pounds, which are integers, and subtracted as such.

Here I say, 3 farthings from 1 farthing I cannot, but, as directed in Rule II. I say,

3 from 4, the number of farthings in 1 penny borrowed,	(10)	(20)	(12)	
and 1 remains; which 1 added to 1 in the major gives 2 farthings for a remainder;	L.	s.	d.	
which I set down, and proceed to the pence, saying, 1	708	14	6 $\frac{1}{4}$	major.
penny borrowed and 10 make 11, which from 6 I cannot, but from 12, the number of pence in 1 shilling, and 1 remains; which 1 added to 6 in the major gives a remainder of 7; which I set down, and go on to the shillings; and because in subtracting shillings we borrow a just number of tens, viz. 2 tens, or 20, I work as directed in Rule III; and in the right-hand column say, 1 borrowed and 7 make 8, which from 4 I cannot, but from 14, and 6 remains; which being set down, I go on to the left-hand column, and say, 1 borrowed and 1 make 2, which from 1 I cannot, but from 2, the number of tens in 1 pound, and 0 remains, which 0 added to 1 in the major gives 1 for a remainder; which I set down, and proceed to the pounds, saying, 1 borrowed and 8 make 9, which from 8 I cannot, but from 18, &c.	278	17	10 $\frac{3}{4}$	minor.
	429	16	7 $\frac{1}{2}$	rem.

penny borrowed and 10 make 11, which from 6 I cannot, but from 12, the number of pence in 1 shilling, and 1 remains; which 1 added to 6 in the major gives a remainder of 7; which I set down, and go on to the shillings; and because in subtracting shillings we borrow a just number of tens, viz. 2 tens, or 20, I work as directed in Rule III; and in the right-hand column say, 1 borrowed and 7 make 8, which from 4 I cannot, but from 14, and 6 remains; which being set down, I go on to the left-hand column, and say, 1 borrowed and 1 make 2, which from 1 I cannot, but from 2, the number of tens in 1 pound, and 0 remains, which 0 added to 1 in the major gives 1 for a remainder; which I set down, and proceed to the pounds, saying, 1 borrowed and 8 make 9, which from 8 I cannot, but from 18, &c.

Note 1. Some add the number borrowed to the figure or number in the major, and then subtract from their sum. Thus, in the farthings they add the 4 borrowed to 1 in the major, and then from the sum 5 they subtract the 3 in the minor; and in the pence they add the 12 borrowed to 6 in the major, and subtract from the sum

sum 18, &c.; but the method taught above is the easiest and most usual.

Note 2. A great many people, in subtracting the shillings, instead of working as directed in Rule III. proceed thus; saying, 1 borrowed and 17 in the minor make 18, which from 14 I cannot, but from 20, the number borrowed, and 2 remains; which 2 added to 14 in the major gives 16 for a remainder. The learner may chuse any of the two methods he likes best.

MORE EXAMPLES.

Ex. I.

	(10)	(20)	(12)
	<i>L.</i>	<i>s.</i>	<i>d.</i>
From	743	16	4
Sub.	375	15	10

Rem.

Ex. 2.

(10)	(20)	(12)
<i>L.</i>	<i>s.</i>	<i>d.</i>
82	13	$4\frac{1}{2}$
54	14	$7\frac{3}{4}$

Ex. 3.

	<i>L.</i>	<i>s.</i>	<i>d.</i>
From	95	8	7
Sub.	68	13	6½

Rem.

Ex. 4.

<i>L.</i>	<i>s.</i>	<i>d.</i>
34	10	6 $\frac{3}{4}$
20	12	7

Ex. 5.

L.

A borrowed of B _____ 150

	<i>L.</i>	<i>s.</i>	<i>d.</i>
A paid B at one time	20	10	6
At another time	18	13	4
At another	12	16	8
Sold him goods to the value of	25	14	7½

In all

Balance due to B

Ex. 6.

Ex. 6.

L.

A lent B 200

		L.	s.	d.
A received at one time	-	36	14	8
At another time	-	40	10	
At another a bill of	-	34	13	4
At another a draught of	-	25	16	10
Goods to the value of	-	16	14	6 $\frac{3}{4}$

In all

Balance due to A

Ex. 7.

A steward collected of rents - L. 2000

Remitted to his master at one time	-	300
At another	-	250
Paid small bills amounting to	-	68 13 4
Paid accounts amounting to	-	45 16 8 $\frac{1}{2}$
Paid of taxes and repairs	-	47 15 6 $\frac{1}{2}$

In all

Balance due to the master

2. AVOIRDUPOIS WEIGHT.

I begin with the pounds, and say, (10) (4) (28)
 24 from 22 I cannot, but from 28, C. *grs.* *lb.*
 the number of pounds in 1 quarter, 84 1 22 major.
 and 4 remains, which added to 22 49 3 24 minor.
 in the major, gives 26 for a remain-
 der; which I set below, and pro- 34 1 26 rem.
 ceed to the quarters, saying, 1 quar-
 ter borrowed and 3 make 4, which from 1 I cannot, but
 from 4, the number of quarters in 1 C. and 0 remains,
 which 0 added to 1 in the major gives 1 for a remainder;
 which I set down, and go on to the C. which are inte-
 gers,

gers, saying, 1 C. borrowed and 9 make 10, which from 4 I cannot, but from 14, &c.

MORE EXAMPLES.

Ex. 1.

	(10)	(20)	(4)	(28)
	<i>T.</i>	<i>C.</i>	<i>qrs.</i>	<i>lb.</i>
From	38	14	2	18
Take	25	14	3	21

Rem.

Ex. 2.

	(10)	(4)	(28)	(16)
	<i>C.</i>	<i>qrs.</i>	<i>lb.</i>	<i>oz.</i>
	48	3	16	10
	23	3	19	13

The *C.* in *Ex. 1.* are subtracted like shillings; that is, either as directed in Rule III. or by the method mentioned in note 2. above.

Ex. 3.

A merchant buys of sugar

<i>C.</i>	<i>qrs.</i>	<i>lb.</i>
80	1	14

Sells at different times

15	2	14
16	2	14
15	3	14
16	3	

Sold in all

Remains on hand

3. TROY WEIGHT.

Ex. 1.

	(10)	(12)	(20)	(24)
	<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>
From	74	10	12	16
Sub.	28	11	15	19

Rem.

Ex. 2.

	(10)	(12)	(20)	(24)
	<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>gr.</i>
	63	8	14	10
	54	10	17	14

Ex.

Ex. 3.

	lb.	oz.	dw.	gr.
Bought of silver	684	10	12	18

Sold at several times

25	8	14	12
14	10	16	10
26	11	18	14
35	10	12	15
42	6	15	17
38	11	14	16

Sold in all

Remains unfold

4. APOTHECARIES WEIGHT.

Ex. 1.

	(10)	(12)	(8)	(3)	(20)
	lb.	$\frac{3}{4}$	3	$\frac{3}{4}$	gr.
From	42	8	4	1	15
Sub.	22	7	5	2	19

Rem.

Ex. 2.

	(10)	(3)	(20)
	3	$\frac{3}{4}$	gr.
	24	1	13
	13	2	17

Ex. 3.

	lb.	$\frac{3}{4}$	3	$\frac{3}{4}$	gr.
From	58	5	3	1	10
Sub.	37	10	5	1	16

Rem.

Ex. 4.

	3	$\frac{3}{4}$	gr.
	37	2	14
	28	2	18

5. WOOL-WEIGHT.

Ex. 1.

	(10)	(12)	(2)	(6 $\frac{1}{2}$)	(2)
	Laft.	fac.	wey.	tod.	ston.
From	84	7	1	3	1
Sub.	73	4	1	4	1

Rem.

Ex. 2.

	(10)	(2)	(7)
	Ston.	clo.	lb.
	25	1	3
	13	1	5

H

6. DRY

6. DRY MEASURE.

Ex. 1.

	(10)	(5)	(8)	(4)
	<i>Load. qrs. bu. pec.</i>			
From	36	3	4	2
Sub.	24	3	6	3

Rem.

Ex. 2.

	(10)	(40)	(8)
	<i>Load. bu. gal.</i>		
	83	25	5
	63	34	6

In subtracting the bushels in Ex. 2. because you borrow a just number of tens, *viz.* 4 tens, or 40, work as directed in Rule III.; that is, proceed with the right-hand column, as in subtraction of integers, and in the left-hand column borrow 4, when the figure in the major is less than the figure to be subtracted.

COAL-MEASURE.

Ex. 1.

	(10)	(4)	(9)
	<i>Chal. qrs. bu.</i>		
From	68	1	3
Sub.	38	2	5

Rem.

Ex. 2.

	(10)	(36)	(4)
	<i>Chal. bu. pec.</i>		
	78	23	1
	49	27	3

SCOTS DRY MEASURE.

Ex. 1.

	(10)	(16)	(4)	(4)
	<i>Chal. bo. fir. pec.</i>			
From	36	11	1	2
Sub.	25	13	2	3

Rem.

Ex. 2.

	(10)	(4)	(4)	(4)
	<i>Bo fir. pec. lip.</i>			
	28	1	2	1
	17	2	3	2

7. WINE.

7. WINE-MEASURE.

Ex. 1.				Ex. 2.		
(10) (2) (2) (63)				(10) (63) (8)		
<i>Tun. pip. bhd. gall.</i>				<i>Hhd. gall. pts.</i>		
From	56	1	1	35	63	45
Sub.	48	1	1	47	38	53
Rem.						

8. BEER and ALE MEASURE.

Ex. 1.				Ex. 2.		
(10) (3) (2) (9)				(10) (3) (36)		
<i>Hhd. kil. firr. gal.</i>				<i>But. bar. gal.</i>		
From	43	1	1	3	57	1
Sub.	35	2	1	5	47	1
Rem.						

SCOTS LIQUID MEASURE.

Ex. 1.				Ex. 2.			
(10) (16) (4) (2)				(10) (2) (2) (4)			
<i>Hhd. gal. qrt. pt.</i>				<i>Pt. cho. mu. gil.</i>			
From	37	11	2	1	47	1	1
Sub.	28	12	2	1	33	1	1
Rem.							

9. CLOTH-MEASURE.

Ex. 1.			Ex. 2.			Ex. 3.		
(10) (4) (4)			(10) (3) (4)			(10) (5) (4)		
<i>Yds. qrs. n.</i>			<i>E. Fl. qr. n.</i>			<i>E. En. qr. n.</i>		
From	38	1	2	48	1	2	78	3
Sub.	23	2	3	29	1	3	57	4
Rem.								

H. 2

10. LONG

10. LONG MEASURE.

Ex. 1.

	(10)	(3)	(8)	(40)
	<i>Lea.</i>	<i>mi.</i>	<i>fur.</i>	<i>po.</i>
From	33	1	4	29
Sub.	25	1	5	37

Rem.

Ex. 2.

	(10)	(5 $\frac{1}{2}$)	(3)	(12)
	<i>Po.</i>	<i>yd.</i>	<i>ft.</i>	<i>inc.</i>
	25	3	1	10
	17	4	2	11

SCOTS LONG MEASURE.

Ex. 1.

	(10)	(8)	(10)	(4)
	<i>Mi.</i>	<i>fur.</i>	<i>ch.</i>	<i>fal.</i>
From	35	3	7	1
Sub.	23	5	7	2

Rem.

Ex. 2.

	(10)	(4)	()	(37)
	<i>Ch.</i>	<i>fal.</i>	<i>el.</i>	<i>inc.</i>
	45	2	3	24
	27	3	5	32

11. LAND-MEASURE.

Ex. 1.

	(10)	(4)	(40)	(30 $\frac{1}{4}$)
	<i>Acr.</i>	<i>roo.</i>	<i>sq. p.</i>	<i>sq. y.</i>
From	38	1	27	17
Sub.	28	3	33	25

Rem.

Ex. 2.

	(10)	(4)	(40)
	<i>Acr.</i>	<i>roo.</i>	<i>sq. p.</i>
	44	2	32
	27	2	36

SCOTS LAND-MEASURE.

Ex. 1.

	(10)	(4)	(40)	(36)
	<i>Acr.</i>	<i>roo.</i>	<i>sq. f.</i>	<i>sq. el.</i>
From	72	2	24	30
Sub.	35	3	27	31

Rem.

Ex. 2.

	(10)	(4)	(40)
	<i>Acr.</i>	<i>roo.</i>	<i>sq. f.</i>
	48	1	33
	28	2	34

12. SUPERFICIAL MEASURE.

Ex. 1.

Ex. 2.

	(10)	(9)	(144)
	Yds.	fee.	inc.
From	723	3	128
Sub.	478	5	132

	(10)	(9)	(144)
	Yds.	fee.	inc.
	654	5	96
	96	7	118

Rem.

When the number of inches in the minor is greater than in the major, subtract, as in integers, from 144, and the difference added to the inches in the major gives the remainder.

13. SOLID MEASURE.

Ex. 1.

Ex. 2.

	(10)	(27)	(1728)
	Yds.	fee.	inc.
From	76	18	378
Sub.	48	21	964

	(10)	(27)	(1728)
	Yds.	fee.	inc.
	48	16	1000
	35	22	1432

Rem.

The last direction takes place here, with this variation, that you subtract the number of inches in the minor from 1728.

14. A CIRCLE.

Ex. 1.

Ex. 2.

	(10)	(12)	(30)	(60)
	Circ.	fig.	°	'
From	13	5	18	46
Sub.	9	7	28	52

	(10)	(30)	(60)	(60)
	Sig.	°	'	"
	7	14	43	50
	4	19	50	54

Rem.

In subtracting the degrees, minutes, and seconds, proceed with the right-hand column as in subtraction of integers, and in the left-hand column of degrees bor-

row

row 3, and in that of minutes and seconds borrow 6, as directed in Rule III.

15. TIME.

Ex. 1.

(10) (13) (4) (7)
Yea. mon. wee. da.

From 24 7 1 4
Sub. 13 10 3 5

Rem.

Ex. 2.

(10) (24) (60) (60)
Da. ho. min. sec.

48 14 36 28
32 18 43 37

III. The Proof of Subtraction.

Merchants and men of business use no other proof besides a revival of the work, or running over it a second time; but it is usual in schools to put the learner upon proving the operation, by some of the three methods following, *viz.*

1. The work may be proved by addition; for if you add the remainder to the minor, the sum, by Axiom X. will be equal to the major, as in the two following examples.

Ex. 1.

5847 major
2569 minor

3278 rem.

5847 proof

Ex. 2.

L. s. d.

73 15 10
48 12 6

25 3 4

73 15 10

2. By subtraction; for if you subtract the remainder from the major, the difference, by axiom X. will be equal to the minor, as follows.

5847

		L.	s.	d.
5847	major	73	15	10
2569	minor	48	12	6
3278	rem.	25	3	4
2569	proof	48	12	6

3. By casting out the 9's; for the major being equal to the sum of the minor and remainder, if you cast the 9's out of the major, and place the excess at the top of the cross, and then cast the 9's out of the minor and remainder, as if they were items in addition, and place the excess at the foot of the cross, it is plain, the figure at the top and foot, if the work be right, will be the same. Only, in proving subtraction of money, Avoirdupois weight, &c. care must be taken to begin with the highest denomination, reducing always the excess to the next inferior denomination, as taught in the proof of addition. See the following examples.

			L.	s.	d.	
6	5847	major	73	15	10	7
X	2569	minor	48	12	6	X
6	3278	rem.	25	3	4	7

IV. Practical Questions.

1. America was discovered by Columbus in the year 1492: How long is it since, this being the year 1763?

Ans. 271 years.

2. The sum of two numbers is 8764; the lesser is 795: What is the greater? *Ans.* 7969.

3. The greater of two numbers is 4832; their difference is 967: What is the lesser? *Ans.* 3865.

4. A borrowed of B L. 347 : 13 : 4, and afterwards paid him L. 218 : 16 : 8. What is the balance due? *Ans.* L. 128 : 16 : 8.

5. What sum added to L. 638 : 14 : 7 $\frac{1}{4}$ will make L. 1000? *Ans.* L. 361 : 5 : 4 $\frac{3}{4}$.

6. A grocer buys C. 17 : 1 : 21 sugar, of which he sells

sells C. 13 : 2 : 24 : How much remains on hand ? *Ans.* C. 3 : 2 : 25.

7. The difference of the weight of two silver bowls is 3 lb. 9 oz. 18 dw. 20 gr. ; the largest bowl weighs 13 lb. 5 oz. 13 dw. 17 gr. : What is the weight of the lesser ? *Ans.* 9 lb. 7 oz. 14 dw. 21 gr.

8. A vintner had in his cellar 18 tuns 2 hhds 48 gallons claret ; but having sold 13 tuns 2 hhds 54 gallons, how much remains ? *Ans.* 4 tuns 3 hhds 57 gallons.

9. A gentleman's estate consisted of 7648 acres 1 rood and 26 poles ; but, to answer the demands of dunning creditors, was obliged to sell off several large inclosures, consisting of 3278 acres 2 roods 32 poles : What is he now possessed of ? *Ans.* 4369 acres 2 roods 34 poles.

10. The planet Venus revolves round the sun in 224 days 16 hours 49 minutes 24 seconds ; and Mercury in 87 days 23 hours 15 minutes 53 seconds : What is the difference of their periodical times ? *Ans.* 136 days 17 hours 33 minutes 31 seconds.

11. A merchant, on balancing his books, finds, that he has in ready money L. 348 : 13 : 4½ ; goods to the value of L. 2635 : 16 : 8¼ ; debts due to him L. 1784, 18 s. 6¾ d. : at the same time he owes to A, L. 275 : 14 : 10 ; and to B, L. 384 : 18 : 7½ : What is his neat stock ? or what will he be worth after all his debts are paid ? *Ans.* L. 4108 : 15 : 2.

12. The weight of two hhds tobacco when packed is C. 9 : 2 : 16 each ; the weight of the two empty hhds is 26½ lb each : What is the neat weight of the tobacco ? *Ans.* C. 18 : 3 : 7.

CHAP. IV.

MULTIPLICATION.

IN multiplication there are two numbers given, *viz.* one to be multiplied, called the *multiplicand* ; and another that multiplies it, called the *multiplier* ; these two go under the common name of *factors* ; and the number

ber arising from the multiplication of the one by the other is called the *product*, and sometimes the *fact*, or the *rectangle*. If a multiplier consists of two or more figures, the numbers arising from the multiplication of these several figures into the multiplicand, are called *particular* or *partial products*; and their sum is called the *total product*.

Multiplication then is the taking or repeating of the multiplicand, as often as the multiplier contains unity. Or,

Multiplication, from a multiplicand and a multiplier given, finds a third number, called the *product*, which contains the multiplicand as often as the multiplier contains unity.

Hence multiplication supplies the place of many additions; for if the multiplicand be repeated or set down as often as there are units in the multiplier, the sum of these, taken by addition, will be equal to the product by multiplication. Thus, $5 \times 3 = 15 = 5 + 5 + 5$.

The first and lowest step in multiplication is, to multiply one digit by another; and the fact or number thence arising is called a *single product*. This elementary step may be learned from the following table, commonly called *Pythagoras's table of multiplication*: which is consulted thus; seek one of the digits or numbers on the head, and the other on the left side, and in the angle of meeting you have their product. The learner, before he proceed further, ought to get the table by heart.

To Pythagoras's table are here added, on account of their usefulness, the products of the numbers 10, 11, 12.

TABLE

T A B L E.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

I. *Multiplication of Integers.*

R U L E S.

I. Set the multiplier below the multiplicand, so as like places may stand under other, viz. units under units, tens under tens, &c.: but if either or both of the factors have ciphers on the right hand, set their first significant figures under other.

The order prescribed in this rule is not absolutely necessary, but very convenient, as will appear in the examples.

II. Beginning at the right hand, multiply each figure of the multiplier into the whole multiplicand, carrying as in addition, and placing the right hand figure of each particular product directly under the multiplying figure. See Axioms II. III. IV. VI.

III. Add the particular products, and their sum will be the total product. See Axiom XII.

EXAMPLE I.

Having placed the multiplier under the multiplicand, as directed in Rule I.

Factors { 94 multiplicand.
7 multiplier.

658 product.

I proceed to the operation, and say, 7 times 4 make 28; I set the 8 below in the place of units, and carry the 2 tens to the next place, as directed in Rule II. saying, 7 times 9 make 63, and 2 that I carried, make 65; I set 5 below in the place of tens, and the 6, which belongs to the next place, I set on its left hand, there being no further place to which it can be carried; so the product is 658.

EXAMPLE II.

Here I first multiply my right-hand figure 8 into the whole multiplicand, as in the former example; then I proceed, and multiply likewise my 6 tens into the whole multiplicand, say-

742 multiplicand.
68 multiplier.

5936 } particular
4452 } products.

50456 Total product.

ing figure, viz in the place of tens, and carry my 1 to the next place, as directed in Rule II. The reason why I set the 2 under the multiplying figure, or in the place of tens, is, because the multiplying figure 6 by Axiom VI. is really 6 tens or 60, and 60 times 2 make 120; so that by carrying the 1 to the next place, and setting down 20, the 0 would fall into the place of units, and throw the 2 into the place of tens; but as 0 can make no alteration in the addition of the partial products, the setting of it down is safely and justly omitted.

From the repetition of the former example on the margin it appears, that though any of the factors may be made the multiplier, the product being the same in both cases, yet the operation becomes easier and shorter by making that factor multiplier which consists of the fewest significant figures. Here likewise observe, that in multiplying 7 hundreds into 8 units of the mul-

I 2

tiplicand,

68
742
136
272
476
50456

tiplicand, I set the right-hand figure of the product under the multiplying figure, *viz.* in the place of hundreds, because the multiplying figure 7 is really 700.

EXAMPLE III.

When the multiplier has ciphers on the right hand, as it would be evidently lost labour to multiply by the ciphers, their only use being to throw figures on their left hand into higher places, I set the first significant figures of the factors under other; and, after the operation is finished, I annex the ciphers of the multiplier to the right hand of the product.

$$\begin{array}{r}
 853 \\
 72000 \\
 \hline
 1706 \\
 5971 \\
 \hline
 61416000
 \end{array}$$

EXAMPLE IV.

When the multiplicand has ciphers on the right hand, the case is in effect the same; wherefore I proceed in the operation as before, and annex the ciphers to the product.

$$\begin{array}{r}
 853000 \\
 72 \\
 \hline
 1706 \\
 5971 \\
 \hline
 61416000
 \end{array}$$

EXAMPLE V.

When both multiplicand and multiplier have ciphers on the right hand; as the case is plainly a compound of the two former, I annex to the product the ciphers of both factors.

$$\begin{array}{r}
 853000 \\
 7200 \\
 \hline
 1706 \\
 5971 \\
 \hline
 6141600000
 \end{array}$$

EXAMPLE VI.

When the multiplier has ciphers intermixed with significant figures, I omit the ciphers, because the multiplying by them would only produce so many lines of ciphers and so be labour in vain; wherefore I multiply by the significant figures only; but I take care to place the right-hand figure of each particular product directly under the multiplying figure.

$$\begin{array}{r}
 29601847 \\
 300905 \\
 \hline
 148009235 \\
 266416623 \\
 88805541 \\
 \hline
 8907343771535 \\
 \text{The}
 \end{array}$$

The reason of setting the right-hand figure of each particular product directly under the multiplying figure, will still further appear by resolving the multiplier into its constituent parts, as in the margin.

$$\begin{array}{r}
 29601847 \text{ multiplicand.} \\
 \hline
 \begin{array}{r}
 5 \\
 900 \\
 300000
 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{partial multipliers.} \\
 \hline
 148009235 \text{ prod. by 5.} \\
 26641662300 \text{ prod. by 900.} \\
 8880554100000 \text{ prod. by 300000.} \\
 \hline
 8907343771535 \text{ total product.}
 \end{array}$$

MORE EXAMPLES.

$$\begin{array}{rcl}
 87694 \times 358 & = & 31394452 \\
 59387 \times 796 & = & 47272052 \\
 78464 \times 4207 & = & 330098048 \\
 978 \times 7800 & = & 7628400 \\
 673000 \times 89 & = & 59897000 \\
 58470 \times 900700 & = & 52663929000
 \end{array}$$

Contractions, and simple ways of working multiplication of integers.

1. To multiply any number by 10, by 100, by 1000, &c. to the given number annex one, two, three ciphers, &c. Thus, $23 \times 10 = 230$; and $384 \times 100 = 38400$; and $745 \times 1000 = 745000$.

2. To multiply any number by 9, by 99, by 999, &c. multiply the given number first by 10, by 100, by 1000, &c. that is, annex one, two, three, &c. ciphers to it; from this subtract the given number, and the remainder is the product; as in the following examples.

Ex. 1.		Ex. 2.		Ex. 3.	
Mult.	47	Mult.	627	Mult.	999
by 9	470	by 99	62700	by 999	999000
Sub.	47	Sub.	627	Sub.	999
Prod.	423	Prod.	62073	Prod.	998001
					From

From Ex. 3. we may learn, in general, that to multiply any number consisting entirely of 9's by itself, is to set 1 in the place of units, then as many ciphers, save one, as there are 9's in the given number; then 8, and on the left hand of 8 as many 9's as there are ciphers on its right.

This method of compendizing may be further extended, thus.

If the multiplier be all 9's, except the right-hand figure, or except the two or the three figures next the right hand, annex as many ciphers to the multiplicand as there are figures in the multiplier; from which subtract the product of the multiplicand into the complement of the right-hand figure to 10, viz. what it wants of 10; or into the complement of the two figures next the right hand to 100; or into the complement of the three figures next the right hand to 1000, &c.

Ex. 1.

$$\begin{array}{r} 7846 \times 996 \\ 7846000 \\ 31384 = 7846 \times 4 \\ \hline 7814616 \text{ prod.} \end{array}$$

Ex. 2.

$$\begin{array}{r} 54786 \times 9988 \\ 547860000 \\ 657432 = 54786 \times 12 \\ \hline 547202568 \text{ prod.} \end{array}$$

Again, if the multiplier be all 9's except the left-hand figure, add unity to the said left-hand figure, and multiply the sum into the multiplicand; to the product annex a cipher for each of the other figures in the multiplier; from which subtract the multiplicand; and the remainder will be the product sought.

Ex. 1.

$$\begin{array}{r} 2846 \times 799 \\ 2846 \\ 8 = 7 + 1 \\ \hline 2276800 \\ 2846 \\ \hline 2273954 \text{ prod.} \end{array}$$

Ex. 2.

$$\begin{array}{r} 4327 \times 1999 \\ 4327 \\ 2 = 1 + 1 \\ \hline 8654000 \\ 4327 \\ \hline 8649673 \text{ prod.} \end{array}$$

3. To

3. To multiply any number by 5; first multiply it by 10, that is, annex a cipher to it, and then halve it: and to multiply any number by 15 use the same method; and add both numbers together, as in the following examples.

Multiply 7439

by 5 — 74390

Product 37195

Multiply 9856

by 15 — 98560
49280 } add

Product 147840

4. To multiply any number by 11, 12, 13, 14, 15, 16, &c. multiply by the unit's figure, and add the back-figure of the multiplicand to the product; and to multiply by 21, 22, 23, 24, 25; 26, 27, &c. add the double of the back-figure; and to multiply by 31, 32, 33, 34, &c. add the triple of it; and to multiply by 112, 113, 114, &c. add the two back figures; and to multiply by 101, 102, 103, 104, &c. add the next back-figure save one: as in the following examples.

Ex. 1.

876 or multiply by
11 11 thus,
876
—
9636

876 or thus, 876
876
—
9636

Ex 2.

694
14
—
9716

Ex. 3.

435
27
—
11745

Ex. 4.

241
34
—
8194

Ex. 5.

7234 or thus, 7234
112 7234
— 7234
810208 7234
— 810208

Ex. 6.

263
119
—
31297

Ex. 7.

745
103
—
76735

In multiplying by 12, as in Ex. 8. it is more usual, and equally easy, to proceed by saying, twelve times 8 make 96, and, setting down the 6, I say, twelve times 4 is 48, and 9 carried is 57; which I set down, and the product is 576.

5. If the multiplier consist of the same figure repeated, as 111, 222, 333, 777, &c. multiply by the unit's figure, and out of that product make up the total product, thus. Begin at the right hand, and first take one figure, then the sum of two, then the sum of three, &c. repeating the operation still from the right hand, as often as there are figures in the multiplier; then, neglecting the right-hand figure, or figure in the first place, take the sum of as many figures toward the left hand as the multiplier has places; and if there be not so many, take the sum of all the figures there are; then, neglecting the figures in the first and second place, begin at the figure in the third place, proceed as before; and thus go on till the last or left-hand figure is taken in alone; as in the following examples.

$$\begin{array}{r} \text{Ex. 1.} \\ 7645 \\ \times 33 \\ \hline \end{array}$$

22935 pr. by 3.

252285 total.

$$\begin{array}{r} \text{Ex. 2.} \\ 4983 \\ \times 666 \\ \hline \end{array}$$

29898 pr. by 6.

3318678 total.

$$\begin{array}{r} \text{Ex. 3.} \\ 38 \\ \times 4444 \\ \hline \end{array}$$

152 pr. by 4.

168872 total.

6. The operation may frequently be rendered shorter or easier, either by addition, subtraction, or a more simple multiplication; and the cases of this kind are so numerous and various, that they admit of no limitation. Consult the following examples and directions.

$$\begin{array}{r} \text{Ex. 1.} \\ 438 \\ + 87 \\ \hline \end{array}$$

$$\begin{array}{r} 3066 \\ 3504 \\ \hline 38106 \end{array}$$

$$\begin{array}{r} \text{Ex. 2.} \\ 374 \\ - 56 \\ \hline \end{array}$$

$$\begin{array}{r} 2244 \\ 1870 \\ \hline 20944 \end{array}$$

$$\begin{array}{r} \text{Ex. 3.} \\ 746 \\ + 84 \\ \hline \end{array}$$

$$\begin{array}{r} 2984 \\ 5968 \\ \hline 62664 \end{array}$$

Ex. 4.

Ex. 4.	Ex. 5.	Ex. 6.
824	685	789
642	147	328
1648	4795	6312
3296	9590	25248
4944		
	100695	258792
529008		

I work the above examples as follows.

Ex. 1. I multiply by 7, and add that product to the multiplicand, instead of multiplying by 8.

Ex. 2. I multiply by 6, and out of that product I subtract the multiplicand, instead of multiplying by 5.

Ex. 3. I multiply by 4, and double that product for 8.

Ex. 4. I multiply by 2; then I double, or multiply that product by 2, for 4; and then add these two products (the right-hand figure of the one to the right-hand figure of the other, &c.), for 6.

Ex. 5. I multiply by 7, and double this product for 14.

Ex. 6. I multiply by 8; and, because 8 times 4 make 32, I multiply that product by 4, for 32.

Several other contractions might be added, but they are rather curious than very useful.

Select methods of multiplying Integers.

1. Instead of multiplying by the multiplier, you may multiply by its component parts, or by the component parts of the nearest composite number; and to or from the last product add or subtract the product of the multiplicand into the difference betwixt the multiplier and the nearest composite number.

Thus, instead of multiplying by 72, you may multiply by 9, and that product by 8; and instead of multiplying by 56, you may multiply by 8 and 7, or by 7, 4, 2; for the choice of the component parts is arbitrary;

K

and

and instead of multiplying by 37, you may multiply by 4 and 9, adding the multiplicand to the last product, for the unit wanting in the product of the component parts; and instead of multiplying by 34, you may multiply by 5 and 7, subtracting the multiplicand from the last product, for the unit of excess in the product of the component parts; and instead of multiplying by 68, you may multiply by 8 and 8, to the last product adding the product of the multiplicand into 4, the difference betwixt the multiplier and the nearest composite number, &c.

Here observe, that when three or more numbers, as in this case, are given to be multiplied into one another, the operation is called *continual multiplication*. And it is of no importance which of the component parts you make the first multiplier; for the last product will be the same, in whatever order the multipliers are taken: but it is convenient that the component parts be all digits, or at least but small numbers, not above 10, 11, or 12. See the following examples.

<i>Ex. 1.</i>		<i>Ex. 2.</i>	
Mult. 436	436	Mult. 351	351
by 9	8	by 8	7
72. ——— or thus, ———	56. ——— or thus, ———		
3924	3488	2808	2457
8	9	7	4
—————	—————	—————	—————
31392	31391	19656	9828
			2
			—————
			19656

Ex. 3.

Multipl.	642	}
by 37.	4	
	2568	
	9	
	—————	
	23112	add
	—————	
	23754	prod.

Ex. 4.

Multipl.	275	}
by 34.	5	
	1375	
	7	
	—————	
	9625	sub.
	—————	
	9350	prod.

Ex.

Ex. 5.
Multipl. 348
by 68.

2784
8

22272 } add
1392 }

23664 prod.

Ex. 6.
Multipl. 324
by 252.

2916
.7

20412
4

81648 prod.

This method may be extended to pretty high numbers. For,

$$10 = 2 \times 5.$$

$$50 = 2 \times 5 \times 5.$$

$$100 = 4 \times 5 \times 5.$$

$$500 = 4 \times 5 \times 5 \times 5.$$

$$1000 = 8 \times 5 \times 5 \times 5.$$

$$1500 = 4 \times 5 \times 5 \times 5 \times 3.$$

$$10000 = 8 \times 5 \times 5 \times 5 \times 2 \times 5.$$

$$100000 = 10 \times 10 \times 10 \times 10 \times 10.$$

And the component parts of the intermediate numbers may easily be discovered. The conveniency and proper use of this method of multiplying will appear in Section 2. following.

2. Instead of beginning with the right-hand figure of the multiplier, you may begin with the left; only take care to place the right-hand figure of every particular product directly under the multiplying figure, as in the following examples.

Ex. 1.
Multipl. 4568
by 234

9136
13704
18272

prod. 1068912

Ex. 2.
Multipl. 6374
by 7408

44618
25496
50992

prod. 47218592

K 2

3. Mul-

3. Multiplication may be performed without any burden to the memory, by setting down every single product, as in the following examples.

Here I say, 8 times 2 is 16, which I set down; then 8 times 4 is 32, which I likewise set down, *viz.* 3 before 1 and 2 under it; then 8 times 7 is 56, I set 5 before 3 and 6 under it; lastly, 8 times 8 is 64, which I set down in the same manner: These added as they stand, give the product.

Ex. 1.
 Mult. 8742
 by 8

 65316
 462

 Prod. 69936

Here observe, that when any single product is under 10, to prevent mistakes by misplacing, you must set a cipher in the place which a second product figure would have possessed.

Ex. 2.
 Mult. 3748
 by 26

 14248
 824
 1016
 648

 Prod. 97448

The disadvantage attending this method of multiplying is, that the addition is tedious.

4. Multiplication may be performed by addition in this manner: Set the digits 1, 2, 3, 4, &c. under one another, and opposite to 1 place your multiplicand; double it for 2; and to this sum add the multiplicand for 3; and again to this sum add the multiplicand for 4: and thus go on till you have a table of the products of the multiplicand by all the digits, or at least as many of them as your multiplier requires; then transfer your particular products out of this table, and their sum will be the total product. This method is convenient in large operations. See the following example.

TABLE

TABLE.

1	3785946
2	7571892
3	11357838
4	15143784
5	18929730
6	22715676
7	26501622
8	30287568
9	34073514

Mult. 3785946
by 659847

26501622
15143784
30287568
34073514
18929730
22715676

Prod. 2498145110262

5. The Noble and ingenious Lord Napier, Baron of Merchiston in Scotland, invented a method of performing multiplication by rods; the separate form of which, with the figures inscribed upon them, are as in the plate, fig. 1.

The rods, excluding the index on the left hand, and the rod of ciphers on the right, are just the several columns of the multiplication-table separated or cut asunder from head to foot; each of the little squares in the table being divided on the rods by diagonal lines into two triangles. The right-hand figure of every single product in the table is placed on the rods in the lower triangle, and the other in the upper: and such products as consist but of one digit are always set in the lower triangle, and the upper one left blank.

The rods are distinguished from one another by their top-figures, 1, 2, 3, 4, 5, &c. making in all ten different rods, besides the index: but as the same figure may occur several times in a multiplicand, it is necessary to have three or four rods of each kind, or to have the four sides of each rod inscribed with a different set of figures.

The rods are fitted for operation thus. To the right side of the index apply a rod on whose top is the left-hand figure of the multiplicand; next to this set the rod on whose top is the following figure of the multiplicand; and

and so on to the right-hand figure: then right against any figure on the index, you have the product arising from the multiplication of that figure into the multiplicand; but to be taken out by the help of an easy addition, as follows.

The figure in the lower triangle on the right-hand rod is the right-hand figure of the product; the figure in the upper triangle on this rod, added to the figure in the lower triangle on the next rod, gives the second figure of the product. Again, the figure in the upper triangle on the second rod, added to the figure in the lower triangle on the rod following, gives the third figure of the product, &c.; and the figure in the upper triangle on the rod next the index, is the last figure of the product. In this manner are the particular products taken from the rods, the sum whereof is the total product. See the plate, fig 2. in which the rods are fitted or set together for the number 9587.

Mult. 9587
by 347

Against 7 on the index I find 67109
Against 4 I find - - 38348
Against 3 I find - - 28761

Total product 3326689

6. If you make a table of the multiplicand for the digits 1, 2, 3, 5, the particular products may be made out from this small table, almost with the same ease, as from the rods, thus.

Suppose for a multiplicand 7894.

TABLE.

1	7894
2	15788
3	23682
5	39470

When the figure of the multiplier is 2, 3, or 5, you have the product by inspecting the table; when the multiplying figure is 4, double the number against 2, or add the numbers against 3 and 1; when it is 6, double the number against 3, or add the

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Napiers Rods

Fig 1.

1	1	2	3	4	5	6	7	8	9	0
2	$\frac{2}{2}$	$\frac{4}{4}$	$\frac{6}{6}$	$\frac{8}{8}$	$\frac{10}{0}$	$\frac{12}{2}$	$\frac{14}{4}$	$\frac{16}{6}$	$\frac{18}{8}$	$\frac{20}{0}$
3	$\frac{3}{3}$	$\frac{6}{6}$	$\frac{9}{9}$	$\frac{12}{2}$	$\frac{15}{5}$	$\frac{18}{8}$	$\frac{21}{1}$	$\frac{24}{4}$	$\frac{27}{7}$	$\frac{30}{0}$
4	$\frac{4}{4}$	$\frac{8}{8}$	$\frac{12}{2}$	$\frac{16}{6}$	$\frac{20}{0}$	$\frac{24}{4}$	$\frac{28}{8}$	$\frac{32}{2}$	$\frac{36}{6}$	$\frac{40}{0}$
5	$\frac{5}{5}$	$\frac{10}{0}$	$\frac{15}{5}$	$\frac{20}{0}$	$\frac{25}{5}$	$\frac{30}{0}$	$\frac{35}{5}$	$\frac{40}{0}$	$\frac{45}{5}$	$\frac{50}{0}$
6	$\frac{6}{6}$	$\frac{12}{2}$	$\frac{18}{6}$	$\frac{24}{4}$	$\frac{30}{0}$	$\frac{36}{6}$	$\frac{42}{2}$	$\frac{48}{8}$	$\frac{54}{4}$	$\frac{60}{0}$
7	$\frac{7}{7}$	$\frac{14}{4}$	$\frac{21}{1}$	$\frac{28}{8}$	$\frac{35}{5}$	$\frac{42}{2}$	$\frac{49}{9}$	$\frac{56}{6}$	$\frac{63}{3}$	$\frac{70}{0}$
8	$\frac{8}{8}$	$\frac{16}{6}$	$\frac{24}{4}$	$\frac{32}{2}$	$\frac{40}{0}$	$\frac{48}{8}$	$\frac{56}{6}$	$\frac{64}{4}$	$\frac{72}{2}$	$\frac{80}{0}$
9	$\frac{9}{9}$	$\frac{18}{8}$	$\frac{27}{7}$	$\frac{36}{6}$	$\frac{45}{5}$	$\frac{54}{4}$	$\frac{63}{3}$	$\frac{72}{2}$	$\frac{81}{1}$	$\frac{90}{0}$

Index

Fig 2.

1	9	5	8	7
2	$\frac{18}{8}$	$\frac{10}{0}$	$\frac{16}{6}$	$\frac{14}{4}$
3	$\frac{27}{7}$	$\frac{15}{5}$	$\frac{24}{4}$	$\frac{21}{1}$
4	$\frac{36}{6}$	$\frac{20}{0}$	$\frac{32}{2}$	$\frac{28}{8}$
5	$\frac{45}{5}$	$\frac{25}{5}$	$\frac{40}{0}$	$\frac{35}{5}$
6	$\frac{54}{4}$	$\frac{30}{0}$	$\frac{48}{8}$	$\frac{42}{2}$
7	$\frac{63}{3}$	$\frac{35}{5}$	$\frac{56}{6}$	$\frac{49}{9}$
8	$\frac{72}{2}$	$\frac{40}{0}$	$\frac{64}{4}$	$\frac{56}{6}$
9	$\frac{81}{1}$	$\frac{45}{5}$	$\frac{72}{2}$	$\frac{63}{3}$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

the numbers against 5 and 1; when it is 7, add the numbers against 5 and 2; when it is 8, add the numbers against 5 and 3; when it is 9, triple the numbers against 3, or add the numbers against 5, 3, and 1.

II. *Multiplication of the parts of Integers.*

Here there are three cases.

1. If your multiplier is a single digit, set it under the units figure of the lowest denomination, multiply it into all the parts of the multiplicand, beginning at the lowest, and carrying always as in addition, or according to the value of the next superior place.

E X A M P L E.

What is the price of 7 packs of cloth, at L. 64, 8 s. 10½ d. *per pack*?

Here I say, 7 times 2 is 14, which is 3 pence and 2 farthings over; I set down the 2 farthings, and carry 3 to the place of pence, saying, 7 times 10 is 70, and 3 that I carried makes 73, which is 6 shillings and 1 penny; I set down the 1 penny, and carry 6 to the place of shillings, saying, 7 times 8 is 56, and 6 that I carried is 62, which makes 3 pounds and 2 shillings; I set down the 2 shillings, and carry 3 to the place of pounds, which are integers.

L.	s.	d.
64	8	10½
		7
451	2	1½

2. If your multiplier consists of two or more figures, multiply continually by its component parts, or by the component parts of the composite number that comes nearest to it; and then multiply the given multiplicand by the difference of the multiplier, and the nearest composite number: the sum or difference of these two products is the answer.

E X A M P L E I.

What is the price of 56 C. tobacco, at L. 2 : 14 : 9¾ *per C.*?

Here the component parts are 8 and 7; for $8 \times 7 = 56$: therefore,

Multiply

Multiply first by 8, and that product by 7; or, which will give the same answer, multiply first by 7, and then that product by 8.

L.	s.	d.
2	14	9 $\frac{3}{4}$
21	18	6
		7
153	9	6

EXAMPLE II.

What is the price of 126 yards of velvet, at L. 3 : 8 : 4 per yard?

Here I multiply first by 6, that product by 7, and that product again by 3: but as the component parts are various, and may be chosen at pleasure, I would have had the same answer, had I multiplied by $9 \times 7 \times 2$; or by $7 \times 3 \times 3 \times 2$.

L.	s.	d.
3	8	4
20	10	
		7
143	10	
		3
430	10	

EXAMPLE III.

What is the price of 67 tuns of iron at L. 18 : 16 : 8 $\frac{1}{2}$ per tun?

Here the nearest composite number is 64, whose component parts are 8×8 ; by which I multiply continually, as in the margin: then I multiply the given multiplicand by 3, the difference betwixt 64 and 67; and because the composite number is less than the multiplier, I add these two products for the answer.

L.	s.	d.
18	16	8 $\frac{1}{2}$
150	13	8
		8
1205	9	4
56	10	1 $\frac{1}{2}$
1261	19	5 $\frac{1}{2}$

EX.

EXAMPLE IV.

What is the weight of 77 chests of goods, each chest weighing C. 2 : 3 : 14 Avoirdupois ?

Here the nearest composite number is 81 ; and accordingly I multiply by its component parts 9×9 : then I multiply the given multiplicand by 4, the difference betwixt 77 and 81 ; and because the composite number is greater than the multiplier, I subtract the one product from the other for the answer.

C.	Q.	lb.
2	3	14
		9
25	3	14
		9
232	3	14
11	2	
221	1	14

} sub.

EXAMPLE V.

What is the price of 274 yards of linen, at 3 s. 4 d. per yard ?

Here I multiply continually by the component parts $10 \times 10 \times 2$, which gives the price of 200 yards : and then, for the price of the rest, I work as follows, viz. for the 70 yards, I multiply the price of 10 yards by 7 ; and for the 4 yards, I multiply the price of one yard by 4 ; and these three products added gives the answer.

L.	s.	d.	
	3	4	price of 1 yard.
		10	
1	13	4	price of 10 yards.
		10	
16	13	4	price of 100 yards.
		2	
33	06	8	price of 200 yards.
11	13	4	price of 70 yards.
	13	4	price of 4 yards.
45	13	4	price of 274 yards.

From the above example may be deduced a general and easy rule for working all questions of this kind ; and is of excellent use when the multiplier happens to be a high number ; viz.

Multiply continually so many times by 10 as there are
L figures

figures in the multiplier, save one; then multiply the given price by the right-hand figure of the multiplier; and again, the first product of 10 by the following figure of the multiplier; and so on, till you have multiplied by all the figures in the multiplier. The sum of these products is the answer.

EXAMPLE VI.

What is the price of 8604 yards of cloth, at 19 s. $6\frac{1}{2}$ d. per yard?

L.	s.	d.	Price of		L.	s.	d.	Price of
19	$6\frac{1}{2}$		1 yd, $\times 4 =$		3	18	2	4 yds.
		10						
9	15	5	10 yds, $\times 0 =$					
		10						
97	14	2	100 yds, $\times 6 =$	586	5			600 yds.
		10						
977	1	8	1000 yds, $\times 8 =$	7816	13	4		8000 yds.
			Price of 8604 yards	8406	16	6		

MORE EXAMPLES.

1. What is the price of 8 yards of cloth, at 13 s. 4 d.? *Ans.* L. 5 : 6 : 8.

2. What comes 12 reams of paper to, at 8 s. $6\frac{1}{2}$ d.? *Ans.* L. 5 : 2 : 6.

3. What cost 96 barrels, at L. 1 : 14 : $7\frac{1}{4}$? *Ans.* L. 166, 2 s.

4. What comes 123 gallons to, at 7 s. $8\frac{3}{4}$ d.? *Ans.* L. 47 : 10 : $8\frac{1}{4}$.

5. A gentleman, whose yearly income is L. 250, spends daily 9 s. $6\frac{1}{2}$ d.: How much does he spend in a year, or 365 days? and how much does he save yearly? *Ans.* He spends L. 174 : 2 : $8\frac{1}{2}$, and saves L. 75 : 17 : $3\frac{1}{2}$.

6. What comes 9760 tuns to, at 18 s. $7\frac{1}{4}$ d.? *Ans.* L. 9078 : 16 : 8.

3. If

3. If your multiplier consists of integers and parts, the operation is performed by a cross multiplication of the several parts of the multiplier into all the parts of the multiplicand.

The contents of mason and joiners work are frequently cast up by this kind of multiplication ; for understanding of which observe, that

The superficial content of any rectangle is found by multiplying the length into the breadth ; and the content of a right-angled triangle is found by multiplying the base into half the perpendicular or height.

The dimensions are usually taken in lineal feet, inches, and lines ; and the operation is performed by the following

R U L E S.

I. Any lineal measure multiplied into the same lineal measure produces squares of that name. Thus, lineal feet multiplied into lineal feet produce square feet ; lineal inches into lineal inches produce square inches, &c.

II. Lineal feet into lineal inches produce rectangles, 1 foot long and 1 inch broad, which divided by 12 quote square feet ; and the remainder multiplied by 12, produces square inches.

III. Lineal feet into lineal lines produce rectangles, 1 foot long and 1 line broad, which divided by 144 quote square feet ; and the remainders are rectangles equal to square inches.

IV. Lineal inches into lineal lines produce small rectangles, 1 inch long and 1 line broad, which divided by 12 quote square inches ; and the remainder, multiplied by 12, produces square lines.

E X A M P L E I.

In an area, pavement, or piece of plaister-work, in length 24 feet 7 inches, and in breadth 18 feet 5 inches, how many square feet ?

F.	in.
24	7
18	5
432	35
20	72
452	107

$18 \times 7 = 126$

$24 \times 5 = 120$

$12)246(20$ by Rule II.

24

$12 \times 6 = 72$

Here I multiply 18 lineal feet into 24 lineal feet, and the product is 432 square feet; then I multiply 5 lineal inches into 7 lineal inches, and the product is 35 square inches, by Rule I.; then I multiply 18 lineal feet into 7 lineal inches, and the product is 126; and again I multiply 24 lineal feet into 5 lineal inches, and the product is 120; which added to the former product gives 246 rectangles, each being 1 foot in length and 1 inch in breadth; these divided by 12 quote 20 square feet; and the remainder 6 multiplied by 12 produces 72 square inches, according to Rule II.; these I add to the former square feet and inches, and find the answer or total product to be 452 square feet; and 107 square inches.

EXAMPLE II.

In an area or floor, in length 38 feet 9 inches 6 lines, and in breadth 23 feet 8 inches 6 lines, how many square feet?

F.	in.	li.
38	9	6
23	8	6
874	72	36
42	84	
278	8	72
919	98	108

$23 \times 9 = 207$

$38 \times 8 = 304$

$12)511(42$

48

31

24

$12 \times 7 = 84$

By Rule II.

$$\begin{array}{r} 38 \times 6 = 228 \\ 23 \times 6 = 138 \\ \hline \end{array}$$

$$\begin{array}{r} 144)366(2 \\ \underline{288} \\ 78 \end{array} \quad \text{By Rule III.}$$

$$\begin{array}{r} 8 \times 6 = 48 \\ 9 \times 6 = 54 \\ \hline \end{array}$$

$$\begin{array}{r} 12)102(8 \\ \underline{96} \\ 12 \times 6 = 72 \end{array} \quad \text{By Rule IV.}$$

Because the sum of the inches exceeds 144, I carry 1 from them to the column of feet, and set down the overplus, *viz.* 98.

The operation may be rendered easier and shorter by previously reducing the factors to two denominations, *viz.* inches and lines. Thus the former example may be proposed and wrought as follows.

In an area or floor, in length 465 inches 6 lines, and in breadth 284 inches 6 lines, how many square inches and feet?

Inch.	lin.
465	6
284	6
132066	36
374	72
132434	108

$$\begin{array}{r} 465 \times 6 = 2790 \\ 284 \times 6 = 1704 \\ \hline \end{array}$$

$$\begin{array}{r} 12)4494(374 \text{ in.} \\ \underline{} \\ 12 \times 6 = 72 \text{ li.} \end{array}$$

by Rule IV.

The answer here is 132434 square inches and 108 square lines; and if the inches be divided by 144, you will have 919 square feet and a remainder of 98 square inches, as before.

Or the factors may be reduced to the lowest denomination, *viz.* lines, and then the product will be square lines, which, divided by 144, will quote square inches, and the remainder will be square lines; and the square inches divided by 144 will quote square feet, and the remainder will be square inches. Again, the square feet divided by 9 will quote square yards, and the remainder will be square feet; and the square yards divided by 36 will quote square roods, and the remainder will be square yards.

If

If this cross multiplication be extended to the mensuration of solids, the content of which is found by multiplying the superficial content of the base into the height, depth, length, or thickness, the operation must be conducted by the following

R U L E S.

V. Any superficial measure multiplied into the same lineal measure produces a solid of the same name. Thus superficial feet multiplied into lineal feet produce solid feet; superficial inches multiplied into lineal inches produce solid inches, &c.

VI. Superficial feet into lineal inches produce parallelopipeds, whose base is 1 square foot, and their height 1 inch; which divided by 12 quote solid feet; and the remainder multiplied by 144 produces solid inches.

VII. Superficial feet into lineal lines produce parallelopipeds, whose base is 1 square foot, and their height 1 line; which divided by 144 quote solid feet; and the remainder multiplied by 12 produces solid inches.

VIII. Superficial inches into lineal lines produce parallelopipeds, whose base is 1 square inch, and their height 1 line; which divided by 12 quote solid inches; and the remainder multiplied by 144 produces solid lines.

IX. Lineal feet into superficial inches produce parallelopipeds, whose base is 1 square inch, and their height 1 foot; which divided by 144 quote solid feet; and the remainder multiplied by 12 produces solid lines.

X. Lineal feet into superficial lines produce parallelopipeds, whose base is 1 square line, and their height 1 foot; which divided by 12 quote solid inches; and the remainder multiplied by 144 produces solid lines.

XI. Lineal inches into superficial lines produce parallelopipeds, whose base is 1 square line, and their height 1 inch; which divided by 144 quote solid inches; and the remainder multiplied by 12 produces solid lines.

E X A M P L E III.

In a piece of timber, whose length is 18 feet 6 inches,
breadth

breadth 2 feet 4 inches, and thickness 2 feet 3 inches, how many solid feet?

F.	in.		
18	6	$2 \times 6 = 12$	
2	4	$18 \times 4 = 72$	
36	24	$12)84(7 \text{ F.}$	by Rule II.
7			
43	24	superficial	
2	3	lineal	
86	72		
10	1296		
	576		
97	216	solid	

$$\begin{array}{l}
 43 \times 3 = 129 \\
 12)129(10 \text{ F.} \\
 144 \times 9 = 1296 \text{ in.}
 \end{array}
 \left. \begin{array}{l} \text{by} \\ \text{Rule} \\ \text{VI.} \end{array} \right\}$$

$$\begin{array}{l}
 2 \times 24 = 48 \\
 12 \times 48 = 576 \text{ in.}
 \end{array}
 \left. \begin{array}{l} \text{by R.} \\ \text{IX.} \end{array} \right\}$$

Here I first multiply 18 feet 6 inches into 2 feet 4 inches, as formerly, and the product is 43 feet 24 inches superficial; which I next multiply into 2 feet 3 inches lineal, thus, 43 superficial feet into 2 lineal feet produce 86 solid feet, and 24 superficial inches into 3 lineal inches produce 72 solid inches, by Rule V.; then 43 superficial feet into 3 lineal inches produce 129 parallelepipeds, whose base is 1 square foot, and their height 1 inch; which divided by 12 quotes 10 solid feet; and the remainder 9 multiplied into 144 produces 1296 solid inches, by Rule VI. Again, 2 lineal feet into 24 superficial inches produce 48; which, being less than 144, I esteem a remainder, and multiplying it into 12 I have a product of 576 solid inches, by Rule IX.

Because the sum of the inches exceeds 1728, I carry 1 from them to the feet, and the overplus 216 I set down.

EXAMPLE IV.

How many solid feet in a polished stone that is 8 feet 9 inches 5 lines long, 7 feet 3 inches broad, and 3 feet 5 lines thick?

F.

F.	in.	l.	$7 \times 9 = 63$	} by Rule II.
8	9	5	$8 \times 3 = 24$	
7	3		—	
56	27		$12)87(7\text{ F.}$	} by Rule III.
7	36		$12 \times 3 = 36\text{ in.}$	
	35		$7 \times 5 = 35\text{ in.}$	
	1	36	$3 \times 5 = 15$	} by Rule IV.
			$12)15(1\text{ in.}$	
63	99	36	sup. $12 \times 3 = 36\text{ li.}$	
3		5	lin.	
189		180	$63 \times 5 = 315$, and $144)315(2\text{ F.}$	} by R. VII.
2	324		$12 \times 27 = 324\text{ in.}$	
2	108		$3 \times 99 = 297$, and $144)297(2\text{ F.}$	} by R. IX.
	9		$12 \times 9 = 108\text{ in.}$	
	41	432	$3 \times 36 = 108$, and $12)108(9\text{ in.}$	} by Rule X.
			$5 \times 99 = 495$, and $12)495(41\text{ in.}$	
193	482	612	solid	} by R. VIII.
			$144 \times 3 = 432\text{ lines.}$	

The operation may be facilitated by previously reducing the three factors to two denominations, *viz.* inches and lines, as was done in Example II. on superficial measure.

Or the three factors may be reduced to the lowest denomination, *viz.* lines, which being multiplied continually, will produce solid lines; which divided by 1728 will quote solid inches, the remainder being solid lines; and the solid inches divided by 1728 will quote solid feet, the remainder being solid inches; and the solid feet divided by 27 will quote solid yards, the remainder being solid feet; and the solid yards divided by 216 will quote solid roods, the remainder being solid yards.

I shall only further observe, that as the rules for working questions by cross multiplication are numerous, and the operation tedious, it is easier to convert the parts into a decimal fraction of their integer, and then work as taught in multiplication of decimals.

III. The Proof of Multiplication.

Multiplication may be proved several ways, *viz.* by multiplication, by division, and by calling out the 9's.

1. By multiplication: Change the places of the factors, and make that the multiplier which before was the multiplicand; and if the work be right, you will have the same product as before; but this method is tedious.

2. By division: When the work is right, the product divided by the multiplier quotes the multiplicand; or, divided by the multiplicand, quotes the multiplier. But this supposes the learner acquainted with division.

3. The most usual method therefore of proving multiplication is by casting out the 9's; which is done thus: Cast the 9's out of the multiplicand and multiplier, and place the excesses on the right and left sides of a cross; multiply these two figures into one another, casting the 9's out of their product, if need be, and place the excess at the top of the cross; then casting the 9's also out of the product of your multiplication, place its excess at the bottom; and if the work be right, the figures at top and bottom will agree, or be the same.

EXAMPLE I.

Here I cast the 9's out of the multiplicand, and place the excess 7 on the right side of the cross; then I cast the 9's out of the multiplier, and place the excess 2 on the left-side of the cross; next I multiply these excesses 2 and 7 into one another, cast the 9's out of their product, and place the excess 5 at the top of the cross; lastly, I cast the 9's out of the product, and place the excess 5 at the foot of the cross: which being the same with the figure at the top, I conclude the work to be right.

754		
38		
—		
6032	2	X
2262		7
—		5
28652		

EXAMPLE II.

Here in casting the 9's out of the multiplicand, and out of the product, I begin with the pounds, and reduce the excess to shillings, and in like manner the excess of the shillings is reduced to pence,

L.	s.	d.	
43	8	4 $\frac{1}{4}$	
—		8	
347	6	10	8
			X
			7

M

and

and that of the pence to farthings. The multiplier being an abstract number, needs no reduction; but if a multiplier be a mixt number, or consist of integers and parts, as feet and inches, &c. the excess of the higher denomination must always be reduced to the lower.

IV. Practical Questions.

1. The continual multiplication of the nine digits will give the number of changes that may be rung on nine bells: How many changes are there? *Ans.* 362880.

2. What is the sum, and what the difference, of six dozen dozen, and half a dozen dozen? *Ans.* Sum 936. Diff. 792.

3. What number taken from the square of 86 will leave 17 times 34? *Ans.* 6818.

4. The lesser of two numbers is 284, their difference is 132; what is the square of their product, and what the cube of their sum? *Ans.* Square of their product is 13958004736; cube of their sum 343000000.

5. Each of nine purses contains L. 81:13:4; how much money in all? *Ans.* 735l.

6. What is the price of 1000 hhds tobacco, at L. 12:3:6 per hhd? *Ans.* 12175l.

7. The wainscoting of a rectangular room measures 156 feet 4 inches about, and is 14 feet 3 inches high; the door is 7 feet by 3 feet 8 inches; eight window-shutters are 7 feet 2 inches by 4 feet 6 inches; the door and window-shutters, being wrought on both sides, are reckoned as work and half; the chimney, 3 feet 9 inches by 3 feet 3 inches, not being inclosed, is to be deducted: How many yards of wainscoting in the room? *Ans.* 261 sq. yards, 8 sq. feet, and 36 sq. inches.

8. How many solid yards of digging in a cellar that is 27 feet 4 inches long, 17 feet 8 inches broad, and 8 feet 2 inches deep? *Ans.* 146 solid yards, 1 solid foot, and 1024 solid inches.

CHAP. V.

DIVISION.

DIVISION discovers how often one number is contained in another: or,

Division, from two numbers given, finds a third, which contains unity as often as the one given number contains the other.

The number to be divided, or which contains the other, is called the *dividend*; the number by which we divide, or which is contained in the dividend, is called the *divisor*; and the number found by division, or which expresses how often the dividend contains the divisor, is called the *quotient* or *quot.*

As multiplication supplies the place of many additions, so division, which is the reverse of multiplication, serves instead of many subtractions; as will thus appear: Suppose it were required to divide 18 by 6, that is, to find how often 6 is contained in 18, the work by subtraction will stand as in the margin:

$$\begin{array}{r} 18 \\ 6 \\ \hline 12 \\ 6 \\ \hline 6 \\ 6 \\ \hline 0 \end{array}$$

I set the divisor on the left of the dividend, leaving room on the right hand for the quotient, as in the margin; and then I say, How often 6 in 18? *Ans.* 3 times: this 3 I set in the quotient; then I multiply the quotient figure 3 into the divisor 6, saying, 3 times 6 make 18; which I set down below the dividend, and subtract it from the dividend, and 0 remains.

$$\begin{array}{r} 6)18(3 \\ 18 \\ \hline 0 \end{array}$$

I. Division of Integers.

R U L E S.

I. From the left-hand part of the dividend point off

M 2

the

the first dividual, viz. so many figures as will contain the divisor.

II. Ask how often the divisor is contained in the dividual, and put the answer in the quotient.

III. Multiply the divisor by the figure set in the quotient, and subtract the product from the dividual.

IV. To the right of the remainder bring down the next figure of the dividend for a new dividual; and then proceed as before.

The substance of these rules is briefly expressed in the following monostich.

Dic quot? multiplica, subduc, transferque sequentem.

First ask how oft? in quot the answer make;

Then multiply, subtract, and down a figure take.

E X A M P L E I.

Here, because the divisor 7 is contained in 8, the left-hand figure of the dividend, I point it off, as my first dividual, according to Rule I.; and then I say, How often 7 in 8? *Anf.* 1 time; which 1 I set in the quotient, as directed in Rule II.; then I multiply the divisor 7 by this quotient figure 1, and subtract the product 7 from the dividual 8, as directed in Rule III.; to the remainder 1 I bring down the following figure of the dividend, for my second dividual, as directed in Rule IV.; then I proceed as before, and say, How often 7 in 17? *Anf.* 2 times; wherefore, setting 2 in the quotient, I multiply and subtract, and find the next remainder to be 3; to which I bring down the following figure of the dividend, and have 35 for my third dividual; then I say, How often 7 in 35? *Anf.* 5 times; which 5 being placed in the quotient, I multiply and subtract, and 0 remains; so the quotient is 125.

Divi-	Divi-	Quo-
for.	dend.	tient.
7)	875	(125
	...	
	7	
	—	
	17	
	14	
	—	
	35	
	35	
	—	
	(0)	

By reviewing the steps of the preceding operation, and reducing, by Axiom VI. the dividuals and quotient-figures

figures to their separate values, the reason of the rules will be obvious; for,

The separate value	7)875(100	
of the first dividual 8	—	
is 800; and the se-	1st dividual 800	20 } partial quot.
parate value of 1, the	700	5 }
first figure put in the	—	
quot, is 100; for	rem. 100	125 total quot.
as 8 contains 7, the	add 70	
divisor, 1 time, so	—	
800 contains it 100	2d dividual 170	
times, and 100 re-	140	
mains; to which I	—	
bring down the fol-	rem. 30	
lowing figure of the	add 5	
dividend 7, whose se-	—	
parate value is 70;	3d dividual 35	
and my second divi-	35	
dual is 170; and as	—	
7 is contained 2 times	(0)	
in 17, so it is con-		

tained 20 times in 170, and 30 remains; to which I bring down the next or last figure of the dividend 5; and my third dividual is 35, in which the divisor 7 is contained 5 times. Now it is evident, that the sum of the partial quots, 125, is the total quot, or a number expressing how often the dividend 875 contains the divisor 7.

From the above example we may learn, that there are always just so many figures in the quotient as there are dividuals; or the first dividual, with the number of subsequent figures in the dividend, is equal to the number of places or figures in the quotient.

Hence likewise may be inferred, that no divisor is contained in any dividual oftener than 9 times; for the dividual, excluding the right-hand figure, is always less than the divisor by 1 at least; and if both be multiplied by 10, or have a cipher annexed to each of them, the product of the dividual will be less than the product of the divisor by 10 at least; but no right-hand figure can supply

supply this defect of 10; therefore the divisor is not contained 10 times in any dividual, and consequently not oftener than 9 times.

Here too observe, that the right-hand figure of the first dividual, and all the subsequent figures of the dividend, have a point or dot set below them, as they are brought down; which is done to prevent mistakes, by distinguishing them, in this manner, from the figures not yet brought down.

EXAMPLE II.

Here, because 8 is not contained in 5, I point off 56 as my first dividual, and say, How often 8 in 56? *Ans.* 7; which I put in the quotient; then I multiply 7 into the divisor 8, and subtract the product 56 from the dividual; and as nothing remains, I bring down the next figure of the dividend, which happens to be a cipher; and as I cannot have 8 in 0, I put 0 in the quotient; and, as multiplying and subtracting is in this case needless, I bring down the next figure of the dividend 3; and as I cannot have 8 in 3, I put another 0 in the quotient, and bring down the next figure of the dividend 2; then I say, How often 8 in 32? *Ans.* 4; which I put in the quotient: then I multiply and subtract; and as nothing remains, I bring down the next figure of the dividend 8, and say, How often 8

$$\begin{array}{r}
 8 \overline{) 56032897} \quad \begin{array}{l} \text{num.} \\ 8 \text{ denom.} \end{array} \\
 \underline{56} \\
 032 \\
 \underline{32} \\
 08 \\
 \underline{8} \\
 9 \\
 \underline{8} \\
 17 \\
 \underline{16} \\
 1
 \end{array}$$

in

in 8? *Ans.* 1; which I put in the quotient: then I multiply and subtract; and as nothing remains, I bring down the next figure of the dividend 9, and say, How often 8 in 9? *Ans.* 1; which I put in the quotient: then I multiply and subtract; and to the remainder 1 I bring down the next and last figure of the dividend 7, and say, How often 8 in 17? *Ans.* 2; which I put in the quotient: then I multiply and subtract, and 1 remains.

To complete the quotient, I draw a line on the right hand, and set the remainder above the line, and the divisor 8 below it, signifying that 1 remains to be divided by 8; or this part of the quotient may be considered as a fraction, whose numerator is 1, and its denominator 8; and the quotient thus completed shews, that the dividend contains the divisor 70041 $\frac{1}{8}$ times, and one eighth part of a time.

Here observe, that not only the last remainder, but every other remainder, must be less than the divisor; for if it be either greater or equal, the divisor might have been oftener got, and the quotient-figure is too little. And should any one in this case attempt to continue the operation, the quotient-figures would all be 9's, the dividuals would prove inexhaustible, and the remainders would constantly increase.

Hence also learn, that if any dividual happen to be less than the divisor, you must put 0 in the quotient, and bring down the next figure of the dividend; and if it be still less than the divisor, you must put another 0 in the quotient, and bring down the following figure of the dividend, &c.

EX-

EXAMPLE III.

Here the divisor consists of 36) 789426 (21928 $\frac{18}{36}$
 two figures; and because it is contained in the two left-hand figures of the dividend 78, I point them off as my first dividual; and say, How often 3 in 7? *Ans.* 2, and 1 remains; which 1 placed, or conceived as placed, on the left hand of the following figure 8, makes 18: then I say, Can I have the following figure of the divisor 6 also 2 times in 18? *Ans.* Yes; consequently I get 36 the divisor 2 times in 78 the dividual; wherefore I put 2 in the quotient, and multiply that 2 into the divisor 36, and the product 72 I subtract from the dividual 78; and to the remainder 6 I bring down the following figure of the dividend 9, for a new dividual: then I say, How often 3 in 6? *Ans.* 2, and 0 remains; again I say, Can I have 6 also 2 times in 9? *Ans.* No; therefore I can have 36 in 69 only 1 time, which 1 I put in the quotient: then I multiply and subtract as before; and to the remainder 33 I bring down the next figure 4 for a new dividual: then, because the dividual consists of a figure more than the divisor, I say, How often the first figure of the divisor 3 in the first two figures of the dividual 33? *Ans.* 9, and 6 remains; which 6 placed on the left hand of the following figure 4 makes 64: again I say, Can I have 6 also 9 times in 64? *Ans.* Yes; consequently 36 can be had 9 times in 334; wherefore I put 9 in the quotient: then I multiply and subtract; and to the remainder 10 I bring down the next figure 2 for a new dividual: here likewise, because the dividual has a figure more than the divisor, I say, How often 3 in 10? *Ans.* 3, and 1 remains; which

$$\begin{array}{r}
 36) 789426 \quad (21928 \frac{18}{36} \\
 \underline{72} \\
 69 \\
 \underline{36} \\
 334 \\
 \underline{324} \\
 102 \\
 \underline{72} \\
 306 \\
 \underline{288} \\
 (18)
 \end{array}$$

1 placed on the left hand of the following figure 2 makes 12: again I say, Can I have 6 also 3 times in 12? *Ans.* No; consequently 36 cannot be had 3 times in 102; wherefore I try if I can have it 2 times; saying, 2 times 3 is 6 from 10, and 4 remains; which 4 placed on the left hand of the next figure 2 makes 42: and I again say, Can I have 6 also 2 times in 42? *Ans.* Yes; consequently 36 can be had 2 times in 102; accordingly I put 2 in the quotient, multiply and subtract; and to the remainder 30 I bring down the next and last figure of the dividend 6, for a new dividual: then, because the dividual has a figure more than the divisor, I say, How often 3 in 30? *Ans.* 9, and 3 remains; which 3 placed on the left hand of the following figure 6 makes 36: and I again say, Can I have 6 also 9 times in 36? *Ans.* No; consequently 36 cannot be had 9 times in 306; therefore I try if it can be had 8 times, saying, 8 times 3 is 24 from 30, and 6 remains; which 6 placed on the left hand of the following figure 6 makes 66: I again say, Can I have 6 also 8 times in 66? *Ans.* Yes; consequently 36 can be had 8 times in 306; wherefore I put 8 in the quotient, and multiply and subtract as before: the last remainder 18 is the numerator of a fraction, and the divisor its denominator, to be annexed to the integral part of the quotient; as was taught in the former example.

The preceding operation points out the manner of procedure when the divisor consists of more figures than one, *viz.* you must take the first figure of the divisor out of the first figure of the dividual, or out of the first two figures of the dividual in case the dividual have a figure more than the divisor: then imagine the remainder to be prefixed to the next figure of the dividual, and try if you can have the second figure of the divisor as often out of this number; if you can, imagine again the remainder to be prefixed to the following figure of the dividual, and try if you can have the third figure of the divisor as often out of this number, &c.; but if you find you cannot have some subsequent figure of the divisor so often as you took the first, you must go back, and take the

N

first

first figure of the divisor 1 time less, or some number of times less, out of the first, or out of the first two figures of the dividend: then proceed as before, repeating the trial, till you find you can have the second, and all the subsequent figures of the divisor, as often as you took the first.

But here observe, that if, in trying how often the divisor can be had in the dividend, either 9, or a number greater than 9, any where remain, you may conclude, without further trial, that all the subsequent figures of the divisor can be had as often as you took the first; as may be thus demonstrated.

Suppose the subsequent figures of the divisor to be the highest possible, that is, all 9's, and the following figures of the dividend the lowest possible, that is, all 0's; again, imagine the remainder 9 prefixed to the following figure of the dividend 0, and it will make 90; now it is plain, that the subsequent figure of the divisor 9 can be had in 90, the highest number of times possible, viz. 9 times, and 9 will remain; which prefixed to the next figure of the dividend 0, makes 90, in which the subsequent figure of the divisor 9 can again be had 9 times, and 9 will remain as before; therefore all the subsequent figures of the divisor can be had as often as you took the first; and if they can be had in this case, much more can they be had when a number greater than 9 remains.

E X A M P L E IV.

Let it be required to divide 170948 l. among 234 men.

Here

Here the divisor consists of three places; and because it is not contained in the three left-hand figures of the dividend, I point off 1709 as my first dividual, and say, How often 2 in 17? *Ans.* 8, and 1 remains; which 1, placed on the left of the following figure 0, makes 10: then I say, Can I have the following figure of the divisor 3 also 8 times in 10? *Ans.* No; consequently 234 cannot be had 8 times in 1709; wherefore I try if I can have it 7 times, saying, 7 times 2 is 14, from 17, and 3 remains; which 3, placed on the left of the following figure 0, makes 30; I say again, Can I have the following figure of the divisor 3 also 7 times in 30? *Ans.* Yes; and because 9 remains, I conclude, without further trial, that the divisor 234 may be had 7 times in the dividual 1709; so I put 7 in the quotient, multiply and subtract, and to the remainder 71 I bring down the following figure of the dividend 4 for a new dividual; and because this dividual consists of the same number of places as the divisor, I say, How often 2 in 7? *Ans.* 3, and 1 remains; which 1, placed on the left of the following figure 1, makes 11; then I say again, Can I have 3 also 3 times in 11? *Ans.* Yes, and 2 remains; which 2, placed on the left of

N 2

$$234)170948(7301.$$

$$\begin{array}{r} 1638 \\ \hline \end{array}$$

$$714$$

$$702$$

$$128 \text{ rem.}$$

$$20$$

$$234)2560(10 \text{ s.}$$

$$234$$

$$220 \text{ rem.}$$

$$12$$

$$440$$

$$220$$

$$234)2640(11 \text{ d.}$$

$$234$$

$$300$$

$$234$$

$$66 \text{ rem.}$$

$$4$$

$$234)264(1\frac{30}{234} \text{ f.}$$

$$234$$

$$(30)$$

the

the following figure 4, makes 24; then I say again, Can I have the third figure of the divisor 4 also 3 times in 24? *Ans.* Yes; and consequently 234 can be had 3 times in 714; wherefore I put 3 in the quotient, multiply and subtract, and to the remainder 12 I bring down the next and last figure of the dividend 8 for a new dividend; but as the divisor cannot be had in this dividend, I put 0 in the quotient, and the dividend 128 becomes the last remainder.

Here, instead of annexing the fraction $\frac{128}{234}$ to the integral part of the quotient, I multiply the 128 l. which remains to be divided among 234 men, by 20, the number of shillings in a pound, and the product 2560 is shillings, which I divide by 234, and the quotient gives 10 s. to each man: the remainder 220 s. I multiply by 12, the number of pence in a shilling, and the product 2640 is pence, which I divide by 234, and the quotient gives 11 d. to each man; the remainder 66 pence I multiply by 4, the number of farthings in a penny, and the product 264 is farthings, which I divide by 234, and the quotient gives to each man 1 farthing, and $\frac{30}{234}$ of a farthing: so each man's share is L. 730 : 10 : 11 $\frac{30}{234}$ f.

EXAMPLE V.

If 34168062 C. of goods be divided into 4875 equal lots, what will be the weight of each lot?

Here

Here the divisor consists of four places; and because it is not contained in the four left-hand figures of the dividend, I point off 34168 as my first dividual, and say, How often 4 in 34? *Ans.* 8, and 2 remains; which 2 placed on the left of the following figure 1, makes 21: then I say, Can I have the following figure of the divisor 8 also 8 times in 21? *Ans.* No; consequently 4875 cannot be had 8 times in 34168; wherefore I try if I can have it 7 times, saying, 7 times 4 is 28, from 34, and 6 remains; which 6, placed on the left of the following figure 1, makes 61: then I say again, Can I have the following figure of the divisor 8 also 7 times in 61? *Ans.* Yes; and 5 remains; which 5, placed on the left of the following figure 6, makes 56: then I say, Can I have the third figure of the divisor 7 also 7 times in 56? *Ans.* Yes, and 7 remains; which 7, placed on the left of the following figure 8, makes 78: then I say, Can I have the fourth figure of the divisor 5 also 7 times in 78? *Ans.* Yes; consequently 4875 can be had 7 times in 34168; wherefore I put 7 in the quotient, multiply and subtract, and to the remainder 43 I bring down the next figure of the dividend 0 for a new dividual; and as I cannot have the divisor 4875 in this dividual 430, I put 0 in the quotient, and bring down the next figure of the dividend 6; but as the divisor 4875 is still greater than the dividual 4306, I put another 0 in the quotient, and bring down the next and last figure of the dividend 2; and as the dividual now consists of five places, and the divisor but of four, I say, How

34125

43062

39000

4062 rem.

4

4875)16248(3 Q.

14625

1623 rem.

28

12984

3246

4875)45444(9¹⁵⁶⁹₄₈₇₅15.

43875

(1569) rem.

How often 4 in 43? *Ans.* 9, and 7 remains; which 7, placed on the left of the following figure 0, makes 70; I say again, Can I have the following figure of the divisor 8 also 9 times in 70? *Ans.* No; consequently 4375 cannot be had 9 times in 43062; wherefore I try if I can have it 8 times, saying, 8 times 4 is 32, which, subtracted from 43, leaves a remainder above 9; therefore I conclude, without further trial, that the divisor can be had 8 times in the dividual; accordingly I put 8 in the quotient, multiply and subtract, and the last remainder is 4062.

Here, as in the former example, instead of annexing the fraction $\frac{4062}{4875}$ to the integral part of the quotient; I multiply the 4062 C. which remains, by 4, the number of quarters in a C. and the product 16248 is quarters; which I divide by 4875, and the quotient gives 3 Q. to each lot: the remainder, *viz.* 1623 Q. I multiply by 28, the number of pounds in a quarter, and the product 45444 is pounds; which I divide by 4875, and the quotient gives to each lot 9 pounds and $\frac{1569}{4875}$ of a pound: so the weight of each lot is 7008 C. 3 Q. 9 $\frac{1569}{4875}$ lb.

E X A M P L E VI.

If, as in the margin, a cipher, or ciphers, possess the right hand of the divisor, cut them off, and cut off, as many figures, *viz.* in this example, the figure 2 from the right hand of the dividend; then divide the remaining figures of the dividend, *viz.* 89678, by the remaining figures of the divisor, *viz.* 648, and you have the integral part of the quotient; but to the remainder 254 annex the figure cut off from the dividend, and you have 2542 for the numerator of your fraction, and the whole divisor 6480 is the denominator.

The reason will appear obvious by working a question in

$$\begin{array}{r}
 648 \overline{) 896782} \\
 \underline{648} \\
 2487 \\
 \underline{1944} \\
 5438 \\
 \underline{5184} \\
 2542
 \end{array}$$

in this manner, and also at full length, without cutting off the cipher or ciphers, and then comparing the two operations.

E X A M P L E VII.

If, as in the margin, the figures cut off from the right hand of the dividend, happen to be all ciphers; in this case, the last remainder, without regarding the ciphers cut off, is the numerator of your fraction, and the significant figures of the divisor the denominator. The reason is assigned in the doctrine of fractions.

$$\begin{array}{r}
 48 \overline{) 00978000} (203\frac{36}{48} \\
 \underline{96} \\
 180 \\
 \underline{144} \\
 (36)
 \end{array}$$

In like manner, if there be cut off from the dividend any number of significant figures, with a cipher or ciphers on their right hand; in this case, the last remainder, with the significant figures cut off, make the numerator of your fraction; and the significant figures of the divisor, with as many ciphers as the number of significant figures cut off from the dividend, make the denominator. Thus, if, in the above example, the figures cut off from the dividend had been 50, the numerator of your fraction would have been 365, and the denominator 480.

M O R E E X A M P L E S.

$$\begin{array}{r|l}
 68 \overline{) 4768943} & 740 \overline{) 638007935} \\
 397 \overline{) 8620094} & 89000 \overline{) 8765432000} \\
 1679 \overline{) 9437856} & 2530000 \overline{) 325643874100}
 \end{array}$$

Contractions, and simple ways of working Division of Integers.

1. To divide any number by 10, 100, 1000, &c. you have only to point off for a remainder as many figures on the right hand of the dividend as the divisor has ciphers, and the other figures on the left of the point or separatrix are the quotient. Thus, 7489634 divided by 10, 100, 1000, &c. stands as follows.

Quot.

Quot. rem.

10)748963.4

100)7489634

1000)7489.634

10000)748.9634

2. If the figures of the divisor are all 9's, or all except the units figure, as 9, 99, 999, 98, 997, 9996, &c. work as follows.

Find a new divisor, by annexing to unity as many ciphers as there are figures in the given divisor, subtract the given from the new divisor, and the remainder or difference is the complement. Divide the given dividend by the new divisor, *viz.* point off so many figures on the right hand as there are ciphers in the said divisor; the figures thus pointed off are to be esteemed a remainder, and the other figures on the left hand are to be accounted a quotient; then multiply this quotient by the complement, placing the units of the product under the units of the former remainder; again, divide this product by the new divisor, by pointing off from the right hand the same number of figures as in the former remainder, and the figures to the left are to be esteemed another quotient; which quotient you are again to multiply by the complement, and divide as before. And in this manner proceed till the last quotient is nothing; then add as in addition of integers, observing the carriage from the left-hand column of the remainders; to the remainders add the product of the said carriage and complement, and the sum is the total remainder; and the sum of the several quotients is the total quotient required.

E X A M P L E I.

Divide 74698 by 98.

New divisor 100

100)746.98

Given divisor 98.

14.92 = 746 × 2

.28 = 14 × 2

Complement 2

Tot. quot. 762.18 + 4 = 22 tot. rem.

Carriage 2 × 2 complement = 4

E X.

E X P L I C A T I O N.

First, to unity I annex two ciphers, because the given divisor consists of two figures, and so the new divisor is 100; from which I subtract the given divisor 98, and there remains 2 for the complement.

Next, I divide the given dividend by the new divisor, *viz.* I point off 98, the two figures next the right hand, for the first remainder; and the figures on the left, namely, 746, is the first quotient.

Then I multiply the said first quotient 746 by the complement 2; and by the new divisor I divide the product 1492, *viz.* I point off 92 for the second remainder, and 14 is the second quotient.

Again, I multiply the second quotient 14 by the complement 2, and the product 28, divided by 100, gives 28 for the third remainder, but nothing to the quotient.

Then I add the several remainders and quotients, and find the total quotient amounts to 762, and the remainders to 18.

Lastly, I multiply 2, the carriage from the left-hand column of the remainders, by the complement 2; and the product 4 I add to the remainders 18, and the sum 22 is the total remainder.

E X A M P L E II.

Divide 849675321 by 994.

New divisor 1000

Given divisor 994

Complement 6

1000)849675.321

5098.050 = 849675 × 6

30.588 = 5098 × 6

.180 = 30 × 6

Total quot. 854804.139 + 6 = 145 tot. rem.

Carriage 1 × 6 Complement = 6.

O

The

The sum of the remainders by itself, or with the product of the complement and carriage, may sometimes happen to be equal to or greater than the given divisor; in which case their difference is the true remainder; and you must add 1 to the sum of the quotients; as in the following

E X A M P L E III.

Divide 4769649 by 997.

New divisor 1000

Given divisor 997

Complement 3

1000) 4769.649

14307 = 4769 × 3

42 = 14 × 3

4783.998 sum of rem.

1.997 given divisor.

Tot. quot. 4784. 1 true rem.

Carriage 0 × 3 Comp. = 0

When the given divisor is all 9's, as 9, 99, 999, 9999, &c. the complement in this case being 1, you have only to transfer the several quotients toward the right hand; as in the following

E X A M P L E IV.

Divide 4379806425 by 9999.

New divisor 10000

Given divisor 9999

Complement 1

10000) 437980.6425

43.7980

43

Quot. 438024.4448 + 1 = 4449 tot. rem.

Carriage 1 × 1 Comp. = 1, to be added to the sum of the remainders.

The reason of the preceding operations is obvious. For the quotient of any number divided by 100, is found by pointing off the two figures next the right hand.

Thus,

Thus, $47856 \div 100$, is 478.56, viz. 478 is the quotient, and 56 the remainder. But 478×99 wants 478 of 478×100 ; wherefore the number 478 on the left of the separatrix is to be considered not only as a partial quotient, but as the difference betwixt the foresaid two products: which difference, being again divided by 100, stands thus, 4.78; but 4×99 wants 4 of 4×100 ; wherefore this 4 must be added to the remainders. Again, the carriage from the remainders not only denotes the number of hundreds carried from them to the partial quotients, but also the excess above the 99's carried thither; wherefore this excess must be added to the remainder, in order to complete it.

Hence, if the divisor be 98, every difference, as well as the carriage, must be doubled; and if the divisor be 97, every difference, as well as the carriage, must be tripled, &c. that is, multiplied by the complement; and if the last remainder be equal to or exceed the given divisor, 1 must be carried from it to the quotient.

For the like reason, if the left-hand figure of the divisor be any digit under 9, and the other figures all 9's, as 79, 899, 6999, &c. you may work as follows.

Add unity to the given divisor, and the sum is the new divisor; by which divide the dividend and the several quotients successively, till nothing remain; then add the remainders and quotients, observing to carry from the remainders the quotient arising from the sum of their left-hand column, divided by the left-hand figure of the new divisor. Here observe, that the left-hand column of the remainders is that which has as many figures or places on its right hand as the new divisor has ciphers.

E X A M P L E I.

Divide 47983756 by 599.

New divisor 600, Complement 1

600) 479837.56

79972.556

$133.172 = 79972 \div 600$

$.133 = 133 \div 600$

Quotient 80106.261 + 1 = 262 tot. rem.

Carriage 1×1 Comp. = 1

O 2

Here

Here the sum of the left-hand column of the remainders is 8; which divided by 6, the left-hand figure of the divisor, gives 1 of carriage, and 2 of a remainder; and the carriage 1×1 comp. gives 1 to be added to the sum of the remainders.

EXAMPLE II.

Divide 1936820 by 3999.

$$4000) 1936. 820$$

$$\underline{484. 820}$$

$$. 484$$

Quotient 484.1304 tot. rem.

Carriage 0×1 Complement $= 0$

3. When the divisor is a single digit, or when it may be reduced to a digit by cutting off ciphers, the operation may be performed mentally; and in this case it is convenient to set the quotient under the dividend, so as its left-hand figure may stand under the right-hand figure of the first dividural; and the remainder may be subjoined by way of fraction to the quotient; as in the following examples.

Ex. 1.

$$2) 568$$

284 quot.

Ex. 2.

$$3) 693$$

231 quot.

Ex. 3.

$$8) 336$$

42 quot.

Ex. 4.

$$4) 672$$

168 quot.

Ex. 5.

$$2) 1783$$

$891\frac{1}{2}$ quot.

Ex. 6.

$$5) 4963$$

$992\frac{3}{5}$ quot.

Ex. 7.

$$2|0) 378|5$$

$189\frac{5}{20}$ quot.

Ex. 8.

$$4|0) 53|7$$

$13\frac{17}{40}$ quot.

Ex. 9.

$$7|00) 269|45$$

$38\frac{345}{700}$ quot.

This

This method may also be profitably used, when the divisor is 11 or 12, or when it is 11 or 12 with a cipher or ciphers annexed.

Ex. 1.

$$\begin{array}{r} 11 \overline{) 561} \\ \hline \end{array}$$

51 quot.

Ex. 2.

$$\begin{array}{r} 11 \overline{) 878} \\ \hline \end{array}$$

79 $\frac{9}{11}$ quot.

Ex. 3.

$$\begin{array}{r} 12 \overline{) 288} \\ \hline \end{array}$$

24 quot.

Ex. 4.

$$\begin{array}{r} 12 \overline{) 342} \\ \hline \end{array}$$

28 $\frac{6}{12}$ quot.

Ex. 5.

$$\begin{array}{r} 11 \overline{) 0748} | 3 \\ \hline \end{array}$$

68 $\frac{3}{110}$ quot.

Ex. 6.

$$\begin{array}{r} 12 \overline{) 004578} \\ \hline \end{array}$$

3 $\frac{978}{1200}$ quot.

4. When the divisor is the product of two or more factors, or component parts, the given dividend may be divided by one of these factors, the quotient by another, the second quotient by a third, &c. till you have divided by every factor; the last quotient is the answer.

E X A M P L E I.

Divide 409248 by 84.

Or,

Because the divisor 84 may be considered as the product of $3 \times 4 \times 7$, I first divide 409248 by 3; and the quotient 136416 I divide by 4; and the second quotient 34104 I divide by 7; and the last quotient 4872 is the answer.

$$\begin{array}{r} 3 \overline{) 409248} \\ \hline \end{array}$$

$$\begin{array}{r} 4 \overline{) 136416} \\ \hline \end{array}$$

$$\begin{array}{r} 7 \overline{) 34104} \\ \hline \end{array}$$

$$\begin{array}{r} 4872 \\ \hline \end{array}$$

$$\begin{array}{l} 12 \times 7 = 84 \\ \text{and therefore} \end{array}$$

$$\begin{array}{r} 12 \overline{) 409248} \\ \hline \end{array}$$

$$\begin{array}{r} 7 \overline{) 34104} \\ \hline \end{array}$$

$$\begin{array}{r} 4872 \\ \hline \end{array}$$

If there be a remainder in each or any of the divisions, the total remainder is found by subtracting the product of the given divisor and last quotient from the given dividend. Or rather let it be obtained thus: Multiply the last remainder into all the preceding divisors continually, adding the several remainders to the products of the divisors to which they belong; and the last product is the total remainder.

E X.

EXAMPLE II.

Divide 234786 by 168.

Here the factors, or component parts of the divisor, may be $4 \times 7 \times 6$; so I divide first by 4, and 2 remains; then I divide by 7, and 1 remains; lastly I divide by 6, and 3 remains.

$$\begin{array}{r}
 4 \overline{) 234786} \quad \text{Rem.} \\
 \underline{0} \\
 7 \overline{) 58696} \quad 2 \\
 \underline{0} \\
 6 \overline{) 8385} \quad 1 \\
 \underline{0} \\
 1397 \quad 3 \\
 \text{Total rem. } 90 \\
 \text{Complete } \left. \begin{array}{l} \\ \\ \end{array} \right\} 1397 \frac{90}{168} \\
 \text{quotient}
 \end{array}$$

Now, to find the total remainder, I multiply the last remainder 3 into the immediately preceding divisor 7; and to the product 21 I add 1, the remainder belonging to that divisor; and the sum 22 I multiply by the first divisor 4; and to the product 88 I add the first remainder 2; and the sum 90 is the total remainder. So the complete quotient is $1397 \frac{90}{168}$.

The reason of collecting the total remainder in this manner is obvious. For the last quotient multiplied back continually into the several divisors, (that is, into the component parts of the given divisor), produces the given dividend, lessened by or wanting the total remainder. And in like manner, the last remainder multiplied back into all the divisors prior to it (taking in the particular remainders that occur in the several divisions) will produce or make up the total remainder.

EXAMPLE III.

Divide 7427 by 24.

Here the component parts of the divisor are $3 \times 4 \times 2$; and as nothing remains in the last division, the total remainder is found by multiplying 3, the last remainder, into 3, the preceding divisor, and to 9, the product, adding 2, the remainder of the first division.

$$\begin{array}{r}
 3 \overline{) 7427} \quad \text{Rem.} \\
 \underline{0} \\
 4 \overline{) 2475} \quad 2 \\
 \underline{0} \\
 2 \overline{) 618} \quad 3 \\
 \underline{0} \\
 \text{Quot. } 309 \\
 \text{Total rem. } 11 \\
 \text{Complete } \left. \begin{array}{l} \\ \\ \end{array} \right\} 309 \frac{11}{24} \\
 \text{quotient}
 \end{array}$$

E X.

EXAMPLE IV.

Divide 20612 by 56.

Here the component parts of the divisor are 7×8 ; and as 4 in the first division happens to be the last and only remainder, there is no prior divisor into which it can be multiplied; so this 4 is the total remainder.

$$\begin{array}{r}
 7 \overline{)20612} \\
 \underline{00} \quad \text{Rem.} \\
 8 \overline{)2944} \quad 4 \\
 \underline{00} \\
 368 \\
 \text{Tot. rem. } 4 \\
 \text{Complete } \left. \begin{array}{l} \text{quotient.} \end{array} \right\} 368 \frac{4}{56}
 \end{array}$$

EXAMPLE V.

Divide 7842 by 42.

Here the component parts of the divisor are 6×7 ; and as 5, the only remainder, is in the last division, the total remainder is found by multiplying it into the preceding divisor 6, and so the fraction is $\frac{30}{42}$. Or rather, when the only remainder happens in the last division, make it the numerator, and the last divisor the denominator; and so the fraction here will be $\frac{5}{7}$, which is equal in value to $\frac{30}{42}$; as will appear in the doctrine of vulgar fractions.

$$\begin{array}{r}
 6 \overline{)7842} \\
 \underline{00} \\
 7 \overline{)1307} \\
 \underline{00} \quad \text{Rem.} \\
 186 \quad 5 \\
 \text{Tot. rem. } 30 \\
 \text{Complete } \left. \begin{array}{l} \text{quotient} \end{array} \right\} 186 \frac{30}{42} \\
 \text{or, } 186 \frac{5}{7}
 \end{array}$$

If the given divisor consist of any figure repeated, as 222, 333, 444, 5555, &c. the component parts may be 111×2 , 111×3 , 111×4 , &c.

EXAMPLE VI.

Divide 78943 by 444.

Here

Were the component parts of the divisor are 111×4 ; and the total remainder is made up as formerly, viz. $3 \times 111 = 333$, and $333 + 22 = 355$.

$$\begin{array}{r}
 4) \\
 111)78943(711 \\
 \dots \text{--- Rem.} \\
 777 \quad 177 \quad 3 \\
 \text{---} \quad \text{Total rem.} \quad 355 \\
 124 \quad \text{Complete} \quad \} \\
 111 \quad \text{quotient} \quad \} 177 \overset{355}{444} \\
 \text{---} \\
 133 \\
 111 \\
 \text{---} \\
 \text{Rem.} \quad 22
 \end{array}$$

5. In division the operation may frequently be rendered more simple, if you divide both divisor and dividend by any number, or numbers, that will divide both without any remainder; and when by this method you can bring them no lower, divide the remaining dividend by the remaining divisor.

EXAMPLE I.

Divide 1692 by 468.

Here I divide both divisor and dividend twice by 2, and again twice by 3; and as I cannot proceed in this manner any further, without a remainder, I divide the remaining dividend 47 by the remaining divisor 13, and the quotient is $3\frac{8}{13}$.

If, by this kind of procedure, the divisor at last dwindle to unity, the last dividend is the quotient.

EXAMPLE II.

Divide 7896 by 84.

Here I divide twice by 2, then by 3, and next by 7; and as the divisor is now reduced to unity, the dividend 94 becomes the quotient.

. This

This method may likewise be used when the dividend, or divisor, or both, are continual products, expressed by the sign of multiplication betwixt the factors. Only observe, that in this case the like number of factors above and below must be divided by the common divisor; and if any factor occur both in the divisor and dividend, it may be dashed in both.

EXAMPLE III.

Divide 12×36 by 18.

In the first step I divide 18 and 12 by 6; in the next I divide 3 and 36 by 3; then I multiply 2 into 12, and the divisor being reduced to unity, the product 24 is the quotient.

$$\begin{array}{c|c|c|c} 12 \times 36 & 2 \times 36 & 2 \times 12 & 24 \\ \hline 18 & 3 & 1 & 1 \end{array}$$

EXAMPLE IV.

Divide $8 \times 72 \times 48$ by $4 \times 24 \times 72$.

Here I first dash the factor 72, both in the divisor and dividend; then I divide the two remaining factors, both above and below, by 4; and proceed to multiply 1 into 6, and 2 into 12, &c.

$$\begin{array}{c|c|c|c|c} 8 \times 72 \times 48 & 8 \times 48 & 2 \times 12 & 24 & 4 \\ \hline 4 \times 24 \times 72 & 4 \times 24 & 1 \times 6 & 6 & 1 \end{array}$$

EXAMPLE V.

Divide $5 \times 49 \times 3 \times 17$ by $17 \times 3 \times 5 \times 35$.

Here I dash the factors $17 \times 3 \times 5$, both in the divisor and dividend; and then divide by 7, &c.

$$\begin{array}{c|c|c|c} 5 \times 49 \times 3 \times 17 & 49 & 7 & 1 \frac{2}{5} \\ \hline 17 \times 3 \times 5 \times 35 & 35 & 5 & \end{array}$$

6. In computations or calculations that require the frequent use of the same divisor, the operation may be rendered more easy and expeditious, by making a table

of the products of the divisor into all the nine digits ; for then you have the quotient-figures, and their products, into the divisor, by inspection.

E X A M P L E.

Divide 47896845 by 748.

748)47896845(64033
.....

TABLE.

1	748
2	1496
3	2244
4	2992
5	3740
6	4488
7	5236
8	5984
9	6732

4488

3016

2992

2484

2244

2405

2244

(161)

Select methods of dividing Integers.

The method of dividing integers hitherto explained is generally used, as being the plainest and best ; but yet a great many other methods are practised ; the chief of which I shall here explain, by working a single example six different ways.

34)72685(2137
....

1. Here the products of the quotient-figures into the divisor are not set down, but subtracted mentally from the dividuuls.

46

128

265

(27)

2. Here

2. Here the quotient is placed above the dividend, and the remainders only are set down; which with the subsequent figures of the dividend make up the several dividends.

$$\begin{array}{r} 2137 \text{ quotient.} \\ 34 \overline{) 72685} \text{ dividend.} \\ \dots \\ 4 \\ 12 \\ 26 \\ (27) \end{array}$$

3. Here the quotient is placed below the dividend, and the several remainders are set above it.

$$\begin{array}{r} 12(2 \\ 426(7 \\ 34 \overline{) 72685} \text{ dividend.} \\ 2137 \text{ quotient.} \end{array}$$

4. Here the products of the quotient-figures into the divisor are set below the dividend, and the several remainders above it.

$$\begin{array}{r} 12(2 \\ 426(7 \\ 34 \overline{) 72685} (2137 \\ 68 \\ 34 \\ 102 \\ 238 \end{array}$$

5. Here too the products of the quotient-figures into the divisor are placed under the dividend, and they, as well as the dividends, are dashed or cancelled as you subtract; which helps to prevent confusion or mistakes in the operation.

$$\begin{array}{r} x \cancel{7}(2 \\ \cancel{4} \cancel{2} \cancel{6}(7 \\ 34 \overline{) \cancel{7} \cancel{2} \cancel{6} \cancel{8} \cancel{5}} (2137 \\ \cancel{6} \cancel{8} \cancel{4} \cancel{2} \cancel{8} \\ \cancel{3} \cancel{6} \cancel{3} \\ \cancel{x} \cancel{7} \end{array}$$

6. Here the divisor is repeated under the dividend; you multiply and subtract at once, and cancel as before.

$$\begin{array}{r} x \cancel{7}(2 \\ \cancel{4} \cancel{2} \cancel{6}(7 \\ \cancel{7} \cancel{2} \cancel{6} \cancel{8} \cancel{5} (2137 \\ \cancel{3} \cancel{4} \cancel{4} \cancel{4} \cancel{4} \\ \cancel{3} \cancel{3} \cancel{3} \end{array}$$

II. Division of the Parts of Integers.

Here there are three cases.

1. If the divisor be a digit, by it divide the integers of the dividend, reduce the remainder to the parts of the next inferior denomination, and add it, when thus reduced, to the said parts; then divide the sum, reducing

cing and adding the remainder to the parts of the following denomination, &c.

Note. If the integral part of the dividend be less than the divisor, you must, in the first place, reduce it to the parts of the next denomination.

E X A M P L E I.

If L. 274 : 13 : 8 : 3 be equally divided among 8 men, what will each man's share be?

Here I first divide the in- $\begin{array}{r} \text{L. } s. \text{ d. } f. \\ 8 \overline{) 274 \text{ } 13 \text{ } 8 \text{ } 3} \end{array}$ dividend.
 tegers L. 274 by 8, and the quotient is L. 34, and L. 2 $\begin{array}{r} 34 \text{ } 6 \text{ } 8 \text{ } 2\frac{3}{8} \end{array}$ quotient.
 remains; which reduced to the next denomination makes 40 shillings; and these added to 13 shillings make 53 shillings; which divided by 8 gives 6 shillings to the quotient, and 5 shillings remains; which 5 shillings reduced make 60 d. and 60 d. added to 8 d. make 68 d.; which divided by 8 gives 8 d. to the quotient, and 4 d. remains, &c.

The operation may, if you please, be drawn out at large; as in the following

E X A M P L E II.

If C. 43 : 2 : 8 of tobacco be made up into 5 equal hhds, what will be the neat weight of each hhd?

Here I divide the C. 43 by $\begin{array}{r} \text{C. } Q. \text{ lb. } C. \text{ } Q. \text{ lb.} \\ 5 \overline{) 43} \end{array}$ by
 5, and the quotient is C. 8, 2 8 (8 2 24
 and C. 3 remains; which re- 40
 duced and added to the 2 Q. —
 makes 14 Q. which I di- 3 rem.
 vide by 5, &c. 4

14
 10
 —
 4 rem.
 28
 —
 120
 10
 —
 20
 20
 —
 (0)

2. If the divisor consists of two or more figures, and be a composite number, resolve it into its component parts, and divide the given dividend by one of these parts, the quotient by another, &c. and the last quotient is the answer.

EXAMPLE I.

If 54 pieces of cloth cost L. 526 : 12 : 5, what is that *per* piece ?

Here the component parts are 6×9 , so I divide by 6, and the quotient by 9; and the last remainder 1 multiplied into the preceding divisor 6, produces 6; to which I add 2, the remainder belonging to that division, and the sum 8 is the total remainder; so the fraction is $\frac{8}{34}$.

	L.	s.	d.	f.	rem.
6)	526	12	5		
9)	87	15	4	3	2
Anf.	9	15	0	$2\frac{8}{34}$	

EXAMPLE II.

If 1 C. or 112 lb of nutmegs be valued at L. 76, 18 s. 8 d. what is that *per* lb ?

Here the component parts are $7 \times 8 \times 2$, by which I divide continually, and make up the total remainder as before.

	L.	s.	d.	f.	rem.
7)	76	18	8		
8)	0	19	9	2	6
2)	1	7	5	2	6
Anf.	13	8	$3\frac{48}{112}$		

But if the divisor be not a composite number, work as in the following

EXAMPLE III.

73 persons go equal shares in an adventure, which cost L. 3648 : 15 : 4 : 2 : What is each person's share ?

L.

L.	s.	d.	f.	L.	s.	d.	f.
73)3648	15	4	2	(49	19	7	3 $\frac{1}{2}$
292	1420	576	276				
728	1435	580	278				
657	73°	511	219				
71 rem.	705	69 rem.	(59)				
20	657	4					
1420 s.	48 rem.	276 f.					
	12						
	96						
	48						
	576 d.						

3. If the divisor consist of integers and parts, reduce both divisor and dividend to the same denomination, and then proceed as in division of integers.

EXAMPLE I.

A nobleman distributes among some poor people L. 20, 5 s.; each person got 7 s. 6 d.: What was the number of the poor?

Here I reduce both the divisor and the dividend to the same denomination, *viz* pence, then I divide, and the quotient 54 is the number of poor people, or it is a number denoting how often 90 d. is contained in 4860 d.

s.	d.	L.	s.
7	6	20	5
12		20	
90 d.		405	
		12	
		810	
		405	

9|0)486|0(
Ans. 54 persons

EXAMPLE II.

The content of a rectangular floor or pavement is 452 square feet, and 107 square inches; the breadth is 18 feet and 5 inches: What is the length?

Here I reduce the dividend to square inches, and the divisor to lineal inches; then I divide, and the quotient 295 is lineal inches, agreeable to the rules laid down in cross multiplication.

F. in.	sq. f.	sq. in.
18 5	452	107
12	144	
—	—	
36	1808	
18	1808	
—	—	
221	452	
	—	in. Fin.

$$\begin{array}{r}
 221 \overline{) 65195} \quad (295 = 24 \text{ } 7 \\
 \underline{442} \\
 2099 \\
 \underline{1989} \\
 1105 \\
 \underline{1105} \\
 0
 \end{array}$$

More examples, in all the parts of division, may be had, by reversing the examples of multiplication.

III. The Proof of Division.

Division may be proved several ways, viz. by multiplication, by division, and by casting out the 9's.

1. By multiplication: Multiply the quotient by the divisor, or the divisor by the quotient; and the product, with the remainder added to it, will be equal to the dividend: Or take the products of the quotient-figures into the divisor, add them in the order they stand under the dividuials; and their sum, with the remainder, will be equal to the dividend.

2. By division: Divide the difference of the dividend and remainder by the quotient, and your next quotient will

will be equal to your first divisor, without any remainder. But this method is tedious.

3. By casting out the 9's : Cast the 9's out of the divisor and quotient, place the excesses on the right and left sides of a cross ; then multiply these two figures into one another, and cast the 9's out of their product ; add the excess to the remainder ; and, casting out the 9's if need be, place the sum or excess at the top of the cross ; then cast the 9's out of the dividend, and set the excess at the bottom. If the work be right, the figures at the top and bottom of the cross will agree, or be the same.

These methods of proof are a proper exercise to the learner in schools ; but, in business, the only proof used is a careful revival of the operation.

IV. *Practical Questions.*

1. What number, multiplied by 74308, will produce 28534272 exactly ? *Ans.* 384.

2. What number, divided by 87424, will quot 4783, and leave a remainder equal to a fourth part of the divisor ? *Ans.* 418170848.

3. The sum of two numbers is 4940, the greater is 4572 : What is the lesser, what their difference, what their product, and what the quotient of the greater divided by the lesser ? *Ans.* 368. 4204. 1682496. $12\frac{156}{368}$.

4. The remainder of a division is 649, the quotient 113, the divisor is the sum of both, and 24 more : What is the dividend ? *Ans.* 89467.

5. There are two numbers, the greater is 24840, which divided by the lesser quotes 72 : What is the lesser, what the difference of their squares, what the square of their sum, and what the cube of their difference ? *Ans.* 345. 616906575. 634284225. 14697221067375.

6. If L. 768 be divided equally among 36 men, what will each man's share be ? *Ans.* L. 21 : 6 : 8.

7. What is the value of 1 yard of cloth, when 84 yards of the same cost L. 36 : 13 : 3 ? *Ans.* 8 s. 8 d. 3 f.

8. A privateer takes a prize to the value of L. 2851, 4 s. ;

of

of which the captain gets $\frac{1}{18}$, each of six officers $\frac{1}{32}$ of the remainder, and the private men, being 45 in number, get the rest equally divided among them: What is each man's share?

		L.	s.	d.
<i>Ans.</i> {	Captain's share	178	4	0
	Each officer's share	83	10	$7\frac{1}{2}$
	Each private man's share	48	5	3

9. Suppose a person in ten years spends of his own L. 2346 : 18 : 6, and of contracted debts to the value of L. 132 : 7 : 8; what is his expence *per year*, *per month*, *per week*, *per day*?

		L.	s.	d.	f.
<i>Ans.</i> {	Per year	247	18	7	$1\frac{6}{10}$
	Per month	20	13	2	$2\frac{56}{120}$
	Per week	4	15	4	$\frac{96}{520}$
	Per day	13	7	$3\frac{84}{30}$	

10. The planet Mercury revolves round the sun in 88 days: How many revolutions will he perform in 17 years and 219 days? *Ans.* 73 revolutions.

11. How many bricks, 9 inches long and 4 inches broad, will floor a room that is 18 feet wide and 24 feet long? *Ans.* 1728 bricks.

12. A rectangular floor contains 919 square feet, 98 square inches, 108 square lines; and the length of it is 38 feet, 9 inches, 6 lines: What is the breadth? *Ans.* 23 feet 8 inches 6 lines.

13. The content of a parallelopiped is 146 solid yards 1 solid foot and 1024 solid inches; the length of it is 27 feet 4 inches; and breadth 17 feet 8 inches: What is the thickness or depth? *Ans.* 8 feet 2 inches.

Q

CHAP.

C H A P. VI.

REDUCTION.

REDUCTION teacheth how to bring a number of one name or denomination to another of the same value; and is either descending, ascending, or mixt.

I. Reduction descending brings a number of a higher denomination to a lower, when the lower is some aliquot part of the higher; as pounds to shillings, pence, or farthings; and is performed by multiplication.

II. Reduction ascending brings a number of a lower denomination to a higher, when the lower is some aliquot part of the higher; as shillings, pence, or farthings, to pounds; and is performed by division.

III. Mixt reduction brings a number of one denomination to another, when the one is no aliquot part of the other; as pounds to guineas; and requires the use of both multiplication and division.

In treating of reduction I shall show its application to money, and to the most usual sorts of weights and measures; in doing of which I shall conjoin reduction descending and ascending, the one serving as a proof of the other; and shall afterwards treat of mixt reduction by itself.

In working reduction, of whatever kind, the following rule is to be observed, *viz.*

Multiply or divide as the tables of coin, weights, and measures, direct.

Reduction descending and ascending.

I: M O N E Y.

Quest. 1. In L. 472 how many shillings, pence, and farthings?

This

This reduction is descending, fo
I multiply the pounds by 20, be-
cause 20 shillings make 1 pound,
and the product is shillings: then
I multiply the shillings by 12, be-
cause 12 pence make 1 shilling, and
the product is pence: lastly, I mul-
tiply the pence by 4, because 4
farthings make 1 penny, and the
product is farthings.

$$\begin{array}{r}
 47210 \text{ pounds.} \\
 \underline{20} \\
 9440 \text{ shillings.} \\
 \underline{12} \\
 1888 \\
 944 \\
 \underline{113280} \text{ pence.} \\
 \underline{4} \\
 453120 \text{ farthings.}
 \end{array}$$

Proof by Reduction ascending.

In 453120 farthings how many pence, shillings, and pounds?

Here I divide the farthings by 4, because 4 farthings make 1 penny, and the quotient is pence: then I divide the pence by 12, because 12 pence make 1 shilling, and the quotient is shillings: lastly, I divide the shillings by 20, because 20 shillings make 1 pound, and the quotient is pounds.

$$\begin{array}{r}
 4)453120 \text{ farthings.} \\
 \underline{} \\
 12)113280 \text{ pence.} \\
 \underline{} \\
 2|0)944|0 \text{ shillings.} \\
 \underline{} \\
 472 \text{ pounds.}
 \end{array}$$

Note 1. To reduce pounds to pence at one operation, multiply by 240, the number of pence in 1 pound.

Note 2. To reduce pounds to farthings at one operation, multiply by 960, the number of farthings in 1 pound.

Note 3. To reduce shillings to farthings at one operation, multiply by 48, the number of farthings in 1 shilling.

I shall here reduce to farthings the L. 472 in Quest. 1. by these notes.

By note 1.

472 pounds.

240

1888

944

113280 pence.

4

453120 farthings.

By note 2.

472 pounds,

960

2832

4248

453120 farthings.

By note 3.

472 pounds,

20

9440 shillings,

48

7552

3776

453120 farthings.

Note 4. To reduce pence to pounds at one operation, divide by 240, the pence in 1 pound.

Note 5. To reduce farthings to pounds at one operation, divide by 960, the farthings in 1 pound.

Note 6. To reduce farthings to shillings at one operation, divide by 48, the farthings in 1 shilling.

Here follows the farthings of Quest. 1. reduced back to pounds by these notes.

By

By note 4.

4)453120 farthings.

24|0)11328|0 d. (472 L.

$$\begin{array}{r} 96 \\ \hline 172 \\ 168 \\ \hline 48 \\ 48 \\ \hline (0) \end{array}$$

By note 5.

96|0)453120(472 L.

$$\begin{array}{r} 384 \\ \hline 691 \\ 672 \\ \hline 192 \\ 192 \\ \hline (0) \end{array}$$

By note 6.

2|0)48)453120(944|0 shillings.

$$\begin{array}{r} 432 \quad 472 \text{ pounds.} \\ \hline 211 \\ 192 \\ \hline 192 \\ 192 \\ \hline (0) \end{array}$$

Quest. 2. In 4386 pounds how many groats?

4386 pounds.
20

87720 shillings.

3 groats in 1 shilling.

Ans. 263160 groats.

Or pounds may be reduced to groats at one operation, if you multiply by 60, the number of groats in 1 pound.

4386 pounds.
60

Ans. 263160 groats.

PROOF.

P R O O F.

In 263160 groats how many pounds ?

3) 263160

Or thus.

610) 2631 610

2|0)8772|0

Ans. L. 4386

***Ans.* L. 4386**

Quest. 3. In 7643 pounds how many three-pences?

7643 pounds.

20

152860 shillings.

4 three-pences in 1 shilling.

Ans. 611440 three-pences.

Or pounds may be reduced to three-pences at one operation, if you multiply by 80, the number of three-pences in 1 pound.

7643 pounds.

80

Ans. 611440 three-pences.

P R O O F.

In 611440 three-pences how many pounds ?

4)611440

Or thus.

8|0)61144|0

210) 1528610

Ans. L. 7643

Anf L. 7643

Quest. 4. In 48 guineas how many shillings, pence, and farthings?

48

48 guineas.

21 the shillings in 1 guinea.

48

96

1008 shillings.

12

12096 pence.

4

48384 farthings.

P R O O F.

In 48384 farthings how many pence, shillings, and guineas?

4)48384 farthings.

12)12096 pence.

21)1008 sh. (48 guineas.

84

168

168

(0)

Quest. 5. In 853 pounds how many crowns, shillings, groats, and pence?

Pounds 853

4 crowns in 1 pound.

Crowns 3412

5 shillings in 1 crown.

Shillings 17060

3 groats in 1 shilling.

Groats 51180

4 pence in 1 groat.

Pence 204720

P R O O F.

P R O O F.

In 204720 pence how many groats, shillings, crowns, and pounds ?

$$\begin{array}{r} 4)204720 \text{ pence.} \\ \hline \end{array}$$

$$\begin{array}{r} 3)51180 \text{ groats.} \\ \hline \end{array}$$

$$\begin{array}{r} 5)17060 \text{ shillings.} \\ \hline \end{array}$$

$$\begin{array}{r} 4)3412 \text{ crowns.} \\ \hline \end{array}$$

$$\begin{array}{r} 853 \text{ pounds.} \\ \hline \end{array}$$

Quest. 6. In L. 458 : 16 : 7 $\frac{3}{4}$ how many shillings, pence, and farthings ?

L. s. d.

$$\begin{array}{r} 458 \quad 16 \quad 7\frac{3}{4} \\ \hline \end{array}$$

20 I take in the 16 s.

Shillings 9176

12 I take in the 7 d.

Pence 110119

4 I take in the 3 f.

Farthings 440479

R R O O F.

In 440479 farthings how many pence, shillings, and pounds ?

$$\begin{array}{r} \text{Rem.} \\ 4)440479 \quad 3 \text{ f.} \\ \hline \end{array}$$

The remainders in reduction ascending are of the same name or denomination with the dividends.

$$\begin{array}{r} 12)110119 \quad 7 \text{ d.} \\ \hline \end{array}$$

$$\begin{array}{r} 2)9176 \quad 16 \text{ s.} \\ \hline \end{array}$$

$$\begin{array}{r} \text{L. } 458 \quad 16 \quad 7\frac{3}{4} \\ \hline \end{array}$$

MORE

MORE EXAMPLES.

Quest. 7. In L. 78342 how many groats and farthings?

Quest. 8. In 69453 guineas how many shillings, three-pences, pence, and farthings?

Quest. 9. In L. 45682 how many crowns, half-crowns, sixpences, and farthings?

Quest. 10. In L. 57843 : 19 : 10½ how many shillings, pence, and halfpence?

2. AVOIRDUPOIS WEIGHT.

Quest. 1. In C. 47 : 1 : 20 how many ounces?

Method 1.

C. Q. lb.

47 1 20

4

189 Q.

28

1512

380

5312 lb.

16

31872

5312

84992 oz.

Method 2.

C. Q. lb.

47 1 20

47

47

47

Q. lb.

48 lb. in 1 20

5312 lb.

16

31872

5312

84992 oz.

Method 3.

C. Q. lb.

47 1 20

564

Q. lb.

48 lb. in 1 20

5312 lb.

16

31872

5312

84992 oz.

R

In

In method 1. in multiplying the C. by 4, I take in the 1 Q.; and in multiplying the Q. by 28, I take in the 20 lb.

In method 2. the operation is the same as multiplying the C. by 112. For 47 placed below 47, with units under units, is the same as multiplying 47 by 2; and what follows is the multiplication of 47 by 11.

In method 3. I multiply the C. by 12; and the product 564, being placed two figures to the right hand of 47, stands so, that being added, their sum will be equal to the product of 47 multiplied by 112. Or, the reason of the operation will still appear plainer by considering, that 47 C. is 4700 lb. and 47 times 12 lb. and therefore to 4700 lb. I add 12 times 47.

P R O O F.

In 84992 ounces how many lb. Q. and C.

	28)	4)	C.	Q.	lb.
16)84992	(5312	(189	(47	1	20
80...	28..	16			
—	—	—			
49	251	29			
48	224	28			
—	—	—			
19	272	(1) Q.			
16	252				
—	—				
32	(20) lb.				
32					
—					
(0)					

M O R E E X A M P L E S.

Quest. 2. In C. 485 : 3 : 24 : 12 oz. how many Q. lb. oz. and dr.?

Quest. 3. In 764 tuns 15 C. 2 Q. 16 lb. how many C. Q. and lb.?

III. T R O Y

3. TROY WEIGHT.

Quest. 1. In 482 lb. 7 oz. 13 dw. 21 gr. how many ounces, penny-weights, and grains?

lb. oz. dw. Gr.

482 7 13 21

12

5791 ounces.

20

115833 penny-weights.

24

463333

231668

2780013 grains.

P R O O F.

In 2780013 grains, how many penny-weights, ounces, and pounds?

2|0)

24)2780013 (11583|3 13 penny-weights.

24.....

12)5791 7 ounces.

38

24

482 pounds.

140

120

200

192

81

lb. oz. dw. gr.

72

In all 482 7 13 21

93

72

Rem. (21) grains.

R 2

MORE

MORE EXAMPLES.

Quest. 2. In 7694 lb. 8 oz. how many grains?

Quest. 3. In 8546 lb. 10 oz. 0 dw. 18 gr. how many grains?

4. DRY MEASURE.

Quest. 1. In 48 loads 12 bushels, how many bushels, pecks, and gallons?

<i>Loads.</i>	<i>Bush.</i>
48	12
40	
1932	bushels.
4	
7728	pecks.
2	
15456	gallons.

P R O O F.

In 15456 gallons, how many pecks, bushels, and loads?

MORE EXAMPLES.

Quest. 2. In 74 bushels 2 pecks, how many pecks, gallons, and pottles?

Quest. 3. In 58 chalders 6 bolls 2 firlots, Scots measure, how many bolls, firlots, and pecks?

5. WINE-MEASURE.

Quest. 1. In 72 hogsheads of wine, how many gallons, and pints?

72 hogheads.

63

216

432

4536 gallons.

8

36288 pints.

P R O O F .

In 36288 pints of wine, how many gallons and hogheads ?

M O R E E X A M P L E S .

Quest. 2. In 45 tuns of wine, how many gallons ?

Quest. 3. In 354 tuns 1 hhd 42 gallons of wine, how many hogheads, gallons, quarts, and pints ?

6. B E E R - M E A S U R E .

Quest. 1. In 54 butts of beer, how many hogheads and gallons ?

54 butts.

2

108 hhds.

54

432

540

5832 gallons.

P R O O F .

In 5832 gallons of wine, how many hogheads and butts ?

M O R E

MORE EXAMPLES.

Quest. 2. In 96 tuns of beer, how many hogheads, gallons, and pints?

Quest. 3. In 25 hogheads of beer, Scots measure, how many gallons, pints, and gills?

7. CLOTH-MEASURE.

Quest. 1. In 34 yards 3 quarters, how many quarters and nails?

$$\begin{array}{r}
 \text{Yds. qrs.} \\
 34 \quad 3 \\
 4 \\
 \hline
 139 \text{ quarters.} \\
 4 \\
 \hline
 556 \text{ nails.}
 \end{array}$$

P R O O F.

In 556 nails, how many quarters and yards?

MORE EXAMPLES.

Quest. 2. In 38 ells Flemish and 2 quarters, how many quarters and nails?

Quest. 3. In 45 ells English and 1 quarter, how many quarters and nails?

8. LONG MEASURE.

Quest. 1. From London to York is 151 miles, how many furlongs, poles, half-yards, inches, and barley-corns, will reach from the one city to the other?

151 miles.

8

1208 furlongs.

40

48320 poles.

11

4832

4832

531520 half-yards.

18

425216

53152

9567360 inches.

3

28702080 barley-corns.

P R O O F .

In 28702080 barley-corns, how many inches, half-yards, poles, furlongs, and miles ?

M O R E E X A M P L E S .

Quest. 2. In 876 miles, how many chains, poles, half-yards, and inches.

Quest. 3. In 950 Scots miles, how many furlongs, chains, feet, and inches ?

9. L A N D - M E A S U R E .

Quest. 1. In 75 acres, how many roods and poles ?

75 acres.

4

300 roods.

40

12000 poles.

Or thus. 75 acres.

160

450

75

12000 poles.

P R O O F .

In 12000 square poles, how many roods and acres?

M O R E E X A M P L E S.

Quest. 2. In 84 acres 3 roods and 25 poles, how many roods and poles?

Quest. 3. In 96 Scots acres, how many roods, falls, and ells?

10. T I M E.

Quest. 1. In 28 years 24 weeks 4 days 16 hours 30 minutes, how many minutes?

<i>Years.</i>	<i>weeks.</i>	<i>days.</i>	<i>hours.</i>	<i>minutes.</i>
28	24	4	16	30
52				
60				
142				
1480 weeks.				
7				
10364 days.				
24				
41462				
20729				
248752 hours.				
60				
14925150 minutes.				

P R O O F.

In 14925150 minutes, how many hours, days, weeks, and years?

The above reduction allows only 364 days to the year; but if you incline to be accurate, find the hours in

in 365 days 6 hours, a complete year, and then reduce as follows.

<i>Days.</i>	<i>hours.</i>	<i>Weeks.</i>	<i>days.</i>	<i>hours.</i>	<i>min.</i>
365	6	24	4	16	30
24		7			
1466		172	days.		
730		24			
8766	hours in one year.	694			
28		345			
70128		4144	hours.		
17532		245448			
245448	h. in 28 years.	249592	hours in all.		
		60			
14975550 minutes.					

P R O O F.

In 14975550 minutes, how many hours and years?

M O R E E X A M P L E S.

Quest. 2. How many hours and minutes from the creation to the birth of Christ, it being accounted 4004 years?

Quest. 3. How many hours and minutes from the birth of Christ to the end of the year 1764?

Mixt Reduction.

In working mixt reduction observe the following

R U L E.

By reduction descending bring the given name to some such third name as is an aliquot part both of the name given and of the name sought; and then by reduction ascending bring the third name to the name sought.

S

Mixt

Mixt reduction, as well as reduction descending and ascending, extends to money, and to things weighed, or measured, as follows.

I. MONEY.

Quest. 1. In 764 l. how many guineas?

Here the name given is	764 pounds.
pounds, the name sought is	20
guineas, and the third name,	
to which the pounds are re-	21) 15280 (727 guineas.
duced, is shillings; for a	147 ..
shilling is an aliquot part	
both of a pound and of a	58
guinea.	42
	160
	147
	(13) shillings.

P R O O F.

In 727 guineas 13 shillings, how many pounds?

<i>Guineas.</i>	<i>shill.</i>
727	13
21	
730	
1455	
210) 15280	
764 pounds.	

Quest. 2. In 832 moidores, how many pounds Sterling?

Here

Here I reduce the moidores to shillings, a shilling being an aliquot part both of a moidore and of a pound Sterling.

832 moidores.
27 shill. in 1 moidore.

5824
1664

2|0)2246|4 shillings.

1123 L. Ster.

P R O O F.

In 1123 l. 4 s. how many moidores ?

L. s.
1123 4
20

27)22464(832 moidores.
21600

86
81

54

54

(0)

Quest. 3. In 968 l. how many merks?

Method 1.

968 l.

3

2) 2904 half-merks.

1452 merks.

Method 2.

968 l.

240

3872

1936

16|0) 23232|0 (1452 merks.

16...

72

64

83

80

32

32

(0)

In method 1. I reduce the pounds to half-merks, a half-merk being an aliquot part both of a pound and of a merk.

In method 2. I reduce the pounds to pence, and then divide by 160, the number of pence in 1 merk; but method 1. is the easier and shorter way.

PROOF.

P R O O F .

In 1452 merks, how many pounds ?

1452 merks.

2

3) 2904 half-merks.

968 l.

Or thus.

1452 merks.

160

8712

1452

24|0)23232|0(968 l.

216

163

144

192

192

(0)

Quest. 4. In 540 dollars, at 4 s. 4 d. per dollar, how many pounds Sterling ?

s. d.

Method 1.

s. d.

Method 2.

4 4

540 dollars

4 4

540 dollars.

12

52

3

13

52 d. in 1 dol. 108

Groats 13 in 1 dol. 162

270

54

24|0)2808|0(117 l.

6|0)702|0

24

117 l.

40

24

168

168

(0)

In

In method 1. I reduce the dollars to pence, and then divide by 240, the number of pence in 1 pound.

In method 2. I reduce the dollars to groats, and then divide by 60, the number of groats in 1 pound.

P R O O F.

In 117 l. Sterling, how many dollars, at 4 s. 4 d. per dollar?

$$\begin{array}{r}
 117 \text{ l.} \\
 \underline{240} \\
 408 \\
 \underline{234} \\
 52)28080(540 \text{ dollars.} \\
 \underline{260} \cdot \cdot \\
 208 \\
 \underline{208} \\
 (0)
 \end{array}$$

$$\begin{array}{r}
 \text{Or thus. } 117 \text{ l.} \\
 \quad \quad 60 \\
 \underline{\quad} \\
 13)7020(540 \text{ dollars.} \\
 \underline{65} \\
 52 \\
 \underline{52} \\
 (0)
 \end{array}$$

Quest. 5. In 4785 l. 13 s. how many pieces of $13\frac{1}{2}$ d. per piece?

$$\begin{array}{r}
 13\frac{1}{2} \text{ d.} \quad \quad \quad \text{L. } 4785 \quad 13 \text{ s.} \\
 \underline{2} \quad \quad \quad \underline{20} \\
 27 \text{ half-pence.} \quad 95713 \text{ shillings.} \\
 \quad \quad \quad \underline{24 \text{ half-pence in 1 shilling.}} \\
 \quad \quad \quad 382852 \\
 \quad \quad \quad \underline{191426} \\
 27)2297112(85078 \text{ pieces of } 13\frac{1}{2} \text{ d.} \\
 \underline{216} \cdot \cdot \cdot \cdot \\
 137 \\
 \underline{135} \\
 211 \\
 \underline{189} \\
 222 \\
 \underline{216}
 \end{array}$$

Rem. (6) half-pence, or 3 d.

P R O O F.

P R O O F.

In 85078 pieces of $13\frac{1}{2}$ d. each, and 3 d. how many pounds?

<i>Pieces.</i>	<i>d.</i>
85078	3
27	
595552	
170156	
	2 0
24) 2297112	(9571 3
216.....	
	L. 4785. 13 s.
137	
120	
171	
168	
31	
24	
72	
72	
(0)	

Quest.

Quest. 6. In L. 872, how many pieces of 6 d. of 5 d. and of 4 d. of each an equal number?

d.	L.
6	872
5	240
4	
	3488
Sum 15 pence.	1744
	15) 209280 (13952 pieces of 6 d. of
	15.... 5 d. and 4 d.
	59
	45
	142
	135
	78
	75
	30
	30
	(0)

P R O O F.

In 13952 pieces of 6 d. of 5 d. of 4 d. of each an equal number, how many pounds?

13952 pieces.	24 0)20928 0(872 L.
15	192..
69760	172
13952	168
Total 209280 pence.	48
	48
	(0)

This

This sort of reduction could easily be extended much further, and might be used to reduce Sterling money to any sort of foreign money, or any sort of foreign money to Sterling; but as this is a wide field, it is more usual, and the better way, to assign a place for this by itself, under the title of *Exchange*. I shall therefore conclude mixt reduction of money by subjoining a few

M O R E E X A M P L E S.

Quest. 7. A gentleman was robbed of 57 guineas 33 half-guineas and 48 moidores, how many pounds Sterling did he lose? *Ans.* L. 141 : 19 : 6.

Quest. 8. In 685 merks 234 nobles and 142 quarter-guineas, how many pounds Sterling? *Ans.* L. 571, 18 s. 10 d.

Quest. 9. In L. 148, how many pieces of $13\frac{1}{2}$ d. of 12 d. of 9 d. of 6 d. of 4 d. and of each an equal number? *Ans.* 798 pieces of each sort, and 9 d. over.

2. A V O I R D U P O I S W E I G H T.

Quest. 1. In 8 bags cotton, each containing C. 12 : 1 : 20, how many tuns?

C.	Q.	lb.	
12	1	20	
		8	
2 0	9 9	1	20
			8

Tuns. C. Q. lb.

(4 19 1 20 *Ans.*

Rem. (19) C.

P R O O F.

If 4 tuns 19 C. 1 Q. 20 lb. of cotton be made up into 8 equal bags, how much will each bag contain?

T

Tuns.

<i>Tuns.</i>	<i>C.</i>	<i>Q.</i>	<i>lb.</i>	<i>C.</i>	<i>Q.</i>	<i>lb.</i>	
8)	4	19	1	20	(	12	1 20 <i>Ans.</i>
	20						
	—						
	99			(5)			
	8			28			
	—			—			
	19			160			
	16			16			
	—			—			
rem.	3			(0)			
	4						
	—						
	13						
	8						
	—						
	(5)						

MORE EXAMPLES.

Quest. 2. In 24 equal cheeses, weighing 3 Q. 15 lb. each, how many C. *Ans.* C. 21 : 0 : 24.

Quest. 3. One buys of a grocer C. 4 : 1 : 14 of pepper, and orders it to be made up into parcels of 14 lb. of 12 lb. of 8 lb. of 6 lb. of 2 lb. and of each an equal number : Required the number of parcels of each sort? *Ans.* 11 parcels of each sort, and 28 pounds over.

3. TROY WEIGHT.

Quest. 1. In 96 pair of silver spurs, weighing 3 oz. 12 dw. *per* pair, how many pounds?

$$\begin{array}{r}
 \text{oz. dw.} \\
 3 \quad 12 \\
 20 \\
 \hline
 72 \text{ dw.} \\
 96 \\
 \hline
 432 \\
 648 \\
 \hline
 2|0)6912 \quad 12 \text{ dw.} \\
 \hline
 12)345 \quad 9 \text{ oz.} \\
 \hline
 28 \text{ lb.} \quad \text{Ans.} \quad 28 \quad 9 \quad 12
 \end{array}$$

P R O O F.

How many pair of spurs, each pair weighing 3 oz. 12 dw. may be made out of 28 lb. 9 oz. 12 dw. of silver?

$$\begin{array}{r}
 \text{oz. dw.} \qquad \text{lb. oz. dw.} \\
 3 \quad 12 \qquad 28 \quad 9 \quad 12 \\
 20 \qquad 12 \\
 \hline
 72 \text{ dw.} \qquad 345 \\
 \qquad 20 \\
 \hline
 72)6912 \text{ (96 pair of spurs Ans.} \\
 \quad 648 \\
 \hline
 \quad 432 \\
 \quad 432 \\
 \hline
 \quad (0)
 \end{array}$$

M O R E E X A M P L E S.

Quest. 2. In 7 dozen silver candlesticks, each weighing 10 oz. 14 dw. how many pounds? *Ans.* 74 lb. 10 oz. 16 dw.

Quest. 3. A merchant sent to a goldsmith 16 ingots of silver.

silver, each weighing 2 lb. 4 oz. and ordered it to be made into bowls of 2 lb. 8 oz. *per* bowl, and tankards of 1 lb. 6 oz. *per* piece, and salts of 10 oz. 10 dw *per* salt, and spoons of 1 oz. 18 dw. *per* spoon, and of each an equal number, how many of each sort will there be? *Ans.* 7 vessels of each sort, and 224 dw. over.

4. WINE-MEASURE.

Quest. 1. In 34 runlets of wine, each runlet containing 18 gallons, how many hogheads?

34 runlets.
18

272

34

63) 612 (9 hhds.
567

Rem. (45) gallons.

Ans. 9 hhds 45 gallons.

P R O O F.

In 9 hhds 45 gallons of wine how many runlets?

Hhds. gall.

9 45
63

18) 612 (34 runlets. *Ans.*

54

72

72

(0)

MORE

M O R E E X A M P L E S.

Quest. 2. In 752 runlets, how many tierce of wine?

Ans. 322 tierce, and 12 gallons.

Quest. 3. In 840 hogsheads, how many puncheons of 80 gallons *per* puncheon? *Ans.* 661 puncheons, and 40 gallons.

5. C L O T H - M E A S U R E.

Quest. 1. In 96 yards, how many ells Flemish?

96 yards.

4

3) 384

Ans. 128 ells Flemish.

P R O O F.

In 128 ells Flemish, how many yards?

M O R E E X A M P L E S.

Quest. 2. In 450 yards, how many ells English? *Ans.*

360.

Quest. 3. In 785 ells Flemish, how many ells English?

Ans. 471.

6. L O N G M E A S U R E.

Quest. 1. In 72 miles, how many furlongs, poles, half-yards, yards, and feet?

72 miles.

8

576 furlongs.

40

23040 poles.

11

2304

2304

2) 253440 half-yards.

126720 yards.

3

380160 feet.

P R O O F.

In 380160 feet, how many yards, half-yards, poles, furlongs, and miles?

M O R E E X A M P L E S.

Quest. 2. Supposing the circumference of the earth to be divided into 360 degrees, and each degree to contain 60 miles, I demand how many miles, furlongs, poles, half-yards, yards, feet, inches, and barley-corns, will reach round the globe of the earth? *Ans.* The barley-corns are 4105728000.

Quest. 3. In 150 English miles, how many Scots miles? *Ans.* 133 miles, 6 furlongs, 10 falls, and 15 feet.

Mixt reduction of weights and measures, as well as that of money, might be carried a great way farther than what is done here; but the above specimens, it is hoped, will be found sufficient instruction to the learner.

Practical Questions.

1. In a purse containing 84 guineas, 56 half-guineas, and

and 48 moidores, how many pounds Sterling? *Ans.* L. 182, 8 s.

2. A gentleman setting out on a journey carried along with him 8 Joannes's, being L. 3, 12 s. each, 17 guineas, 13 half guineas, 14 crowns, 10 half-crowns, 15 shillings, and 9 sixpences; and finds, on his return, by his pocket-book, that he had spent L. 49, 4 s.: How many pounds Sterling did he carry out, and how many brought he home? *Ans.* He carried out L. 59, 4 s. and brought home L. 10.

3. A merchant bought 30 hogheads sugar, viz. 12 hhds containing C 16 : 1 : 14 each, 12 hhds weighing C. 14 : 3 : 7 each, and 6 hhds containing C. 15 : 2 : 21 each: How many tuns of sugar did he purchase? *Ans.* 23 tuns 8 C. 1 Q. 14 lb.

4. In 96 firkins of beer, 9 gallons each, how many hhds of 54 gallons *per* hhd? *Ans.* 16 hhds.

5. A piece of ground, consisting of 27 acres, is to be laid out in small divisions, of 3 roods and 10 poles each; how many small divisions will there be? *Ans.* 32.

6. Queen Elisabeth came to the throne of England the 17th of November 1558, and died the 24th of March 1603, in the 70th year of her age: What year was she born? and how many months, of 28 days each, did she reign, reckoning 365 days 6 hours to a year? *Ans.* She was born in the year 1533, and reigned 578 months 2 weeks and 1 day.

C H A P. VII.

THE RULE OF THREE.

THE Rule of Three, called also, on account of its excellence, the *Golden Rule*, from certain numbers given, finds another; and is divided into simple and compound, or into single and double.

SECTION I

The simple or single Rule of Three.

THE simple rule of three, from three numbers given, finds a fourth, to which the third bears the same proportion as the first does to the second.

The nature and properties of proportional numbers may be understood sufficiently for our purpose from the following observations.

In comparing any two numbers, with respect to the proportion which the one bears to the other, the first number, or that which bears proportion, is called the *antecedent*; and the other, to which it bears proportion, is called the *consequent*; and the quantity of the proportion or ratio is estimated by the quot arising from dividing the antecedent by the consequent. Thus the ratio or proportion betwixt 6 and 3 is the quot arising from dividing the antecedent 6 by the consequent 3; namely, 2; and the ratio or proportion betwixt 1 and 2 is the quot arising from the division of the antecedent 1 by the consequent 2; namely $\frac{1}{2}$, or one half.

Four numbers are said to be proportional when the ratio of the first to the second is the same as that of the third to the fourth; and the proportional numbers are usually distinguished from one another as in the following examples.

$$4 : 2 :: 16 : 8. \qquad 6 : 9 :: 12 : 18.$$

Proportional numbers, or numbers in proportion, are usually denominated *terms*; of which the first and last are called *extremes*, and the intermediate ones get the name of *means*, or *middle terms*.

If four numbers are proportional, they will also be inversely proportional; that is, the first consequent will be to its own antecedent as the second consequent is to its antecedent; or the fourth term will be to the third as the second is to the first. Thus, if $6 : 3 :: 10 : 5$, then by inversion, $3 : 6 :: 5 : 10$, or $5 : 10 :: 3 : 6$.

Euclid

Euclid v. 4. cor. By either of these kinds of inversion may any question in the rule of three be proved.

If four numbers are proportional, they will also be alternately proportional; that is, the first antecedent will be to the second antecedent as the first consequent is to the second consequent; or the first term will be to the third term as the second term is to the fourth. Thus, if $8 : 4 :: 24 : 12$, then, by alternation, $8 : 24 :: 4 : 12$. *Euclid* v. 16.

But the celebrated property of four proportional numbers is, that the product of the extremes is equal to the product of the means. Thus, if $2 : 3 :: 6 : 9$, then $2 \times 9 = 3 \times 6 = 18$. *Euclid* vi. 16.

Hence we have an easy method of finding a fourth proportional to three numbers given, *viz.*

Multiply the middle number by the last, and divide the product by the first, the quot gives the fourth proportional.

E X A M P L E.

Given 6, 5, and 36, to find a fourth proportional; put x equal to the fourth proportional, then $6 : 5 :: 36 : x$, and $5 \times 36 = 180 = 6 \times x$; wherefore, dividing the product 180 by the factor 6, the quot gives the other factor x , namely 30, the fourth proportional sought.

Every question in the rule of three may be divided into two parts, *viz.* a supposition and a demand; and of the three given numbers, two are always found in the supposition, and only one in the demand.

E X A M P L E.

If 4 yards cost 12 shillings, what will 6 yards cost at that rate?

In this question the supposition is, if 4 yards cost 12 shillings; and the two terms contained in it are 4 yards and 12 shillings; the demand lies in these words, What will 6 yards cost? and the only term found in it is 6 yards.

The supposition and demand being thus distinguished,
U
proceed

proceed to state the question, or to put the terms in due order for operation, as the following rules direct.

R U L E I.

Place that term of the supposition, which is of the same kind with the number sought, in the middle. The two remaining terms are extremes, and always of the same kind.

R U L E II.

Consider, from the nature of the question, whether the answer must be greater or less than the middle term; and if the answer must be greater, the least extreme is the divisor; but if the answer must be less than the middle term, the greatest extreme is the divisor.

R U L E III.

Place the divisor on the left hand, and the other extreme on the right; then multiply the second and third terms, and divide their product by the first; and the quotient gives the answer; which is always of the same name with the middle term.

When the divisor happens to be the extreme found in the supposition, the proportion is called *direct*; but when the divisor happens to be the extreme in the demand, the proportion is *inverse*.

The three rules delivered above are indeed so framed, as to preclude the distinction of direct and inverse, or render it needless, the left-hand term being always the divisor; but yet the direct questions being plainer in their own nature, and more easily comprehended by a learner, I shall, in the first place, exemplify the rules by a set of questions of the direct kind, and shall afterwards adduce a collection of such as are inverse.

I. *The simple Rule of Three direct.*

Quest. 1 If 4 yards cost 12 shillings, what will 6 yards cost at that rate?

The supposition and demand of this question have already been distinguished, and the two terms in the former

former are 4 yards 12 shillings, and the only term in the latter is 6 yards.

The number sought is the price of 6 yards, and the term in the supposition of the same kind is the price of 4 yards, *viz.* 12 shillings, which I place in the middle, as directed in Rule I. and the two remaining terms are extremes, and of the same kind, *viz.* both lengths.

It is easy to perceive that the answer must be greater than the middle term; for 6 yards will cost more than 4 yards; therefore the least extreme, *viz.* 4 yards, is the divisor, according to Rule II.

$$\begin{array}{rcl}
 & \text{yds.} & \text{s.} & \text{yds.} \\
 \text{If } 4 & : & 12 & :: 6 \\
 & & 6 & \\
 & & \text{—} & \\
 & 4) & 72 & (18 \text{ shillings } \textit{Ans.} \\
 & & 4 & \\
 & & \text{—} & \\
 & & 32 & \\
 & & 32 & \\
 & & \text{—} & \\
 & & (0) &
 \end{array}$$

Wherefore I place the divisor 4 yards on the left hand, and the other extreme 6 yards on the right; and multiplying the second and third terms, I divide their product by the first term, and the quot 18 is the answer, and of the same name with the middle term, *viz.* shillings, according to Rule III.

And because the divisor is the extreme found in the supposition, the proportion is direct.

Quest. 2. If 7 C. of pepper cost 21 l. how much will 5 C. cost at that rate?

The supposition in this question is, that 7 C. of pepper costs 16 l. and the two terms in it are 7 C. and 16 l.; the demand is, How much will 5 C. cost? and the term in it is 5 C.

The number sought is the price of 5 C. and the term in the supposition of the same kind is the price of 7 C. *viz.* 21 l. which I place in the middle. The two remaining terms are extremes, and of the same kind, *viz.* quantities of pepper.

It is obvious, that the answer must be less than the middle term; for 5 C will cost less than 7 C.; and therefore the greatest extreme, *viz.* 7 C. is the divisor.

G.	L.	C.
If 7 :	21 ::	5
	5	
7)	105	(15 l. <i>Ans.</i>
	7	
	35	
	35	
	(0)	

Accordingly I place the divisor 7 C. on the left hand, and the other extreme 5 C. on the right; and having multiplied the second and third terms, I divide their product by the first term, and the quot 15 is the answer, of the same name with the middle term, *viz.* L. Sterling.

And because the divisor happens to be the extreme in the supposition, the proportion is direct.

Quest. 3. If 13 yards of velvet cost L. 21, what will 27 yards cost at that rate?

r. L. r.
If 13 : 21 :: 27 * Rem. 4 s.

27	12
117	13) 48 (3 d.
42	39

When there happens to be a remainder, it may be reduced to the next inferior denomination, and the operation continued, as in the margin; and in this case the quot will consist of two or more parts.

13) 567 (43 L.	Rem. 9 d.
52	4
47	13) 36 (2 f.
39	26

Rem. 8 L.	Rem. 10 f.
20	

13) 160 (12 s.

13	L. s. d. f.
30	Ans. 43 12 3 2 $\frac{10}{13}$
26	

* Rem. 4 s.

Such remainders are always of the same name with the preceding part of the quot. Thus, the first remainder 8, and the first part of the quot 43, are both pounds; and the second remainder 4, and the second part of the quot 12, are both shillings; and the third remainder 9, and the third part of the quot 3, are both pence; and the fourth remainder 10, and the fourth part of the quot 2, are both farthings.

As we have no money under farthings, the last remainder cannot be reduced any lower; so there remains 10 farthings to be divided by 13; that is, there is wanting to complete the quot the thirteenth part of 10 farthings, or the thirteenth part of every remaining farthing; that is, ten thirteen parts of one farthing; so I set the remainder 10 above and the divisor 13 below a line drawn between them, in the form of a fraction, of which
the

the remainder is the numerator and the divisor the denominator.

Quest. 4. If 14 lb. of tobacco cost 27 shillings, what will 478 lb. cost at that rate?

$$\begin{array}{r}
 \text{lb. s. lb.} \\
 \text{If } 14 : 27 :: 478 \\
 \hline
 27 \\
 \hline
 3346 \\
 956 \\
 \hline
 14) 12906 \quad \begin{array}{l} 2|0 \\ 92|1 \end{array} \\
 12600 \\
 \hline
 \text{L. } 46 \quad 1 \\
 30 \\
 28 \\
 \hline
 26 \\
 14 \\
 \hline
 \text{Rem. } 12 \text{ s.} \\
 12 \\
 \hline
 14) 144 \quad (10 \text{ d.} \\
 14 \\
 \hline
 \text{Rem. } 4 \text{ d.} \\
 4 \\
 \hline
 14) 16 \quad (1 \text{ f.} \\
 14 \\
 \hline
 \text{Rem. } 2 \text{ f.}
 \end{array}$$

Here the middle term being shillings, the first part of the quot is also shillings; which I divide by 20, and so reduce it to pounds.

There remains at last 2 farthings, to be divided by 14; so I subjoin these by way of fraction to the preceding part of the quot; as in the former example.

$$\begin{array}{r}
 \text{L. s. d. f.} \\
 \text{Ans. } 46 \quad 1 \quad 10 \quad 1\frac{2}{14}
 \end{array}$$

uest.

Quest. 5. If 15 ounces of silver be worth L. 3, 15 s.
What are 86 ounces worth at that rate?

If the middle term be complex, or mixt, and so consist of two or more parts, reduce it to the lowest, and the answer will come out of the same name with that of the lowest part.

Thus,

The middle term here is complex, consisting of two parts, *viz.* pounds and shillings; so I reduce it to shillings, and the quot comes out in shillings; which I reduce to pounds.

oz. L. s. oz.
If 15 : 3—15 :: 86

20

75 shill.

86

450

600

2|0)

15) 6450 (43|0 shillings.

6000

L. 21 10

45

45

(0)

L. s.

Ans 21 10

The parts of a complex term may be distinguished from one another by a stroke; as in this example.

Quest.

Quest. 6. If 18 C. sugar cost L. 54, what will 7 C. 3 Q. 14 lb. cost at that rate?

C.	L.	C.	Q.	lb.
If 18	: 54 ::	7	— 3	— 14
18		7		
18		7		
18		798		
2016 lb.		882 lb.		

When any or both of the extremes happen to be complex, or mixt, and so consist of two or more parts, reduce both extremes to the lowest of the parts; for in the operation both extremes must be equally low, or of the same name.

Thus, in this example the extreme on the right hand is complex, consisting of three parts, *viz* C Q and lb.; and so I reduce both this extreme, and also the other extreme on the left hand, to lb. the lowest of the parts.

54
3528
4410
2016) 47628 (23 L.
4032
7308
6048

Rem. 1260 L.

20

2016) 25200 (12 s.

2016

5040

4032

Rem. 1008 s.

12

2016) 12096 (6 d.

12096

(0)

L. s. d.
Anf. 23 12 6

Quest.

Quest. 7. If C. 3 : 1 : 14 of raisins cost L. 10 : 2 : 6, what will 6 C. 3 Q. cost at that rate ?

C.	Q.	lb.	L.	s.	d.	C.	Q.
If 3	1	14	10	2	6	6	3
3			20			6	
3						6	
342			202			684	
			12				
378	lb.					756	lb.
			2430	d.			
			756				
			1458				
			1215				
			1701				

Here all the terms are complex, and the two extremes are both reduced to lb. that being the lowest of the parts.

The middle term, for the like reason, is reduced to pence, and accordingly the quot or answer comes out in pence.

378	1837080	(4860	(405
1512	...	48	..
3250		60	
3024		60	
2268		(0)	
2268			
(0)			

L. 20 5

L. s.
Ans. 20 5

Quest. 8. A goldsmith bought a wedge of gold, which weighed 14 lb. 3 oz. 8 dw. for the sum of L. 514, 4 s. : I demand what it stood him *per* ounce ?

X

lb.

lb. oz. dw. lb. s. oz.
 If 14—3—8 : 514—4 :: 1
 12 20 20

171 oz. 10284 shill. 20 dw.
 20 20

dw. 3428 3428) 205680 (60 shill.
 20568
 ——— L. 3
 (0)

Ans. L. 3 per ounce.

Here the left-hand extreme being complex, both it and the other extreme are reduced to penny-weights, the lowest of the parts.

The middle term, being likewise complex, is reduced to shillings; and so the answer comes out in shillings; which I reduce to pounds.

Quest. 9. A grocer bought 4 hogsheds of sugar, each weighing neat C. 6 : 2 : 14, which cost him L. 2 : 8 : 6 per C.: Required the value of the 4 hogsheds at that rate?

C.	Q.	lb.
6	2	14
		4

26	2
26	
26	
2656	
2968	lb. in 4 hhds.

lb. L. s. d. lb.
If 112 : 2—8—6 :: 2968

20	582
48	5936
12	23744
	14840

582 d. 112) 1727376 (15423 d. 3 Rem.

112	1727376	(15423 d. 3
112		
	2 0)128 5	
607		
560	L. 64 5 3	Ans.

When part of a term only is given, and the term to be completed by multiplication, it is generally convenient to complete the term, or even to reduce it, before you state the question.

473
4,8

257
224

336
336

(0)

Quest. 10. If 4 s. 6 d. is the price of 1 yard, how many yards will L. 16, 4 s. buy?

X 2

If

$$\begin{array}{rcl} \text{s.} & \text{d.} & \text{yd.} \\ \text{If } 4-6 : 1 :: 16-4 & & \\ \hline 2 & & 20 \end{array}$$

When the middle term happens to be 1, or unity, in this case, as 1 neither multiplies nor divides, you have only to divide the third term by the first, and the quot is the answer.

$$\begin{array}{r} 9 \overline{) 648} \quad (72 \text{ y.} \\ \underline{63} \\ 18 \\ \underline{18} \\ (0) \end{array}$$

Ans. 72 yards.

In this example both extremes are complex, and reduced to sixpences.

Quest. 11. If the yearly rent of a house be L. 73, how much is that *per* day?

Days. *L.* *day.*

$$365 : 73 :: 1$$

When the third term happens to be 1, you have only to divide the middle term by the first, and the quot is the answer.

$$\begin{array}{r} 20 \\ \hline 365 \overline{) 1460} \quad (4 \text{ s.} \\ \underline{1460} \\ (0) \end{array}$$

Ans. 4 s. *per* day.

But if in this case the middle term be less than the first, it must be reduced to some lower denomination, *viz.* till the middle term thus reduced may be divided by the first.

Quest. 12. A merchant bought 14 pieces of broad cloth, each piece containing 28 yards, at 13 s. 6½ d. *per* yard: How much did the 14 pieces cost?

114 pieces. *yd* *s.* *d.* *yd*
 28 If 1 : 13—6½ :: 392
 — 12 325
 112 — 1960
 28 — 784
 392 yds in 14 p. — 1176
 325 — 210)
 24) 127400) 530|8 s.
 120... —
 — L. 265 8
 74
 72
 —
 200
 192
 —
 Rem. (8) half-pence.
 L. s. d.
 Ans. 265 8 4

When the first term happens to be 1, the product of the second and third is the answer

The middle term here being complex, is reduced to halfpence; and accordingly the product, or answer, comes out in halfpence, which I reduce to pounds.

Quest. 13 A draper buys 50 pieces of kersey, each piece containing 34 ells Flemish, to pay at the rate of 8 s. 4 d. *per* ell English: How much did the 50 pieces cost?

grs. *s.* *d.* *grs.*
 If 5 : 8—4 :: 5100
 12 100
 — —
 100 5) 510000
 — —
 12) 102000
 — —
 210) 850|0
 — —
 Ans. 425 l.

34
 50
 —
 1700 ells Flemish
 3
 —
 5100 quarters.

Quest.

Quest. 14. What is the yearly interest of L. 78, 16 s. 6 d. at 5 per cent.?

$$\begin{array}{r} \text{pr.} \\ \text{If } 100 : 5 :: 78-16-6 \\ \hline 5 \end{array}$$

$$\begin{array}{r} \text{Int.} \\ \text{1|00)3|94-2-6(3 } 18 \text{ } 9 \text{ } 3\frac{6}{10} \text{ Ans.} \\ \hline 20 \\ \hline 18|82 \\ \hline 12 \\ \hline 9|90 \\ \hline 4 \\ \hline 3|60 \end{array}$$

MORE EXAMPLES.

Quest. 15. If 17 yards of cloth cost L. 19 : 2 : 6, what will 35 yards cost at that rate? **Ans.** L. 39 : 7 : 6.

Quest. 16. If 24 lb. of raisins cost 6 s. 6 d. what will 18 frails cost, each frail weighing 3 Q 18 lb? *Ans.* L. 24 : 17 : 3.

Quest. 17. If 40 pieces of cloth cost 750 l. 15 s. what is the price of 1 piece? *Ans.* L. 18 : 15 : 4½.

Quest. 18. If 1 ell of cloth cost 8 s. 4 d. how many ells will L. 10 : 16 : 8 buy? *Ans* 26 ells.

Quest. 19. A gentleman expends daily L. 5, 15 s.
How much is that in a year? *Ans.* L. 2098, 15 s.

Quest. 20. A merchant buys cloth to the value of L. 537, 12 s. at 5 s. 4 d. *per* yard: How many yards did he buy? *Ans.* 2016 yards.

Quest. 21. If I sell 14 yards for L. 10, 10 s. how many ells Flemish shall I sell for L. 283 : 17 : 6, at that rate? *Ans.* 504 $\frac{2}{3}$ ells Flemish.

Quest. 22. What is the price of 14 ingots of silver, each ingot weighing 7 lb. 5 oz. 10 dw. at 5 s. per ounce? *Ans.* L. 313, 5 s.

Quest. 23. A grocer mingles 17 C. 3 Q. 14 lb. of sugar,

gar, at L. 4, 4 s. *per* C. with 8 C. 3 Q. 21 lb. other sugar, at 6 d. *per* lb.: What is 1 C. of this mixture worth? *Ans.* L. 3 : 14 : 8.

Quest. 24. A draper buys 8 packs of cloth, each pack containing 4 parcels, and each parcel 10 pieces, and each piece 26 yards, for which he paid at the rate of L. 4, 16 s. for 6 yards: How much did the whole cost? *Ans.* L. 6656.

Quest. 25. A merchant bought 242 yards of broad cloth, which cost him in all L. 209, 1 s.; for 96 yards of which he paid at the rate of 13 s. 4 d. *per* yard: How much did he give *per* yard for the rest? *Ans.* 18 s. 6 d. *per* yard.

Quest. 26. An oilman bought 3 tuns of oil at the rate of L. 50 : 11 : 4 *per* tun; and so it happened that it leaked out 85 gallons; but he is minded to sell it again, so as to be no loser: How must he sell it *per* gallon? *Ans.* At 4 s. 6 $\frac{1}{8}$ $\frac{1}{4}$ d. *per* gallon.

Quest. 27. A shopkeeper bought 4 bales of cloth, each bale containing 6 pieces, and each piece 27 yards, at L. 16, 4 s. *per* piece: What was the price of the whole? and what the rate *per* yard? *Ans.* The whole cost L. 388, 16 s. and the rate *per* yard was 12 s.

Quest. 28. A bankrupt, who owed L. 1490 : 5 : 10, compounds with his creditors for 12 s. 6 d. *per* L.: How much did he pay in all? *Ans.* L. 931 : 8 : 7 $\frac{1}{4}$.

Quest. 29. If the rent of a small estate be L. 250, what will this afford to spend every calendar month, in order to save, or lay up, L. 80 at the year's end? *Ans.* L. 14 : 3 : 4.

Quest. 30. The rents of a whole parish amount to L. 3500, and a rate is granted of L. 131, 5 s.: What is that *per* pound? *Ans.* 9 d.

Quest. 31. Received 42 C. 2 Q. cotton, at L. 3, 15 s. *per* C. and 12 lb. cloves, at 9 s. 1 d. *per* lb.; for which I have given in part 1000 yards linen, at 2 s. 9 d. *per* yard: What balance do I owe? *Ans.* L. 27 : 6 : 6.

Quest. 32. A chapman bought a parcel of serge and shalloon, which together cost him L. 61 : 9 : 2; the quantity of serge he bought was 236 yards, at 3 s. 4 d. *per*

per yard; and for every 2 yards of serge he had 3 yards of shalloon: How much shalloon had he? and what did it cost him *per yard*? *Ans.* 354 yards of shalloon, at 15 d. *per yard*.

II. The simple Rule of Three inverse.

Quest. 1. If 8 men can do a piece of work in 12 days, in how many days will 16 men do the same?

In this question the supposition is, If 8 men do a piece of work in 12 days, and the two terms contained in it are 8 men and 12 days: The demand lies in these words, In how many days will 16 men do the same? and the only term contained in it is 16 men.

The number sought here is the days in which 16 men will do the work, and the term in the supposition of the same kind is 12 days; wherefore I place 12 days as the middle term, according to Rule I. the two remaining terms are extremes, and of the same kind, *viz.* both of them men.

It is obvious that the answer must be less than the middle term; for 16 men will do the work in fewer days than 8 men; and therefore, by Rule II. the greatest extreme, *viz.* 16 is the divisor; which I place on the left hand, and the other extreme on the right, as directed in Rule III. Then multiplying the second and third, and dividing their product by the first, the quotient comes out in days; that is, of the same name with the middle term.

And because the extreme found in the demand happens to be the divisor, the proportion is inverse.

Quest. 2. If 30 yards of cloth that is 5 quarters broad, be required to hang a bed, how many yards of 3 quarters broad will serve the same purpose?

The supposition here is, that 30 yards of cloth 5 quarters broad are sufficient to hang a bed, and the two terms

men. days. men.

16 : 12 :: 8

8

—

16)96(6 days. *Ans.*

96

—

(6)

terms in it are 30 yards of length, and 5 quarters of breadth: The demand is, How many yards of cloth that is 3 quarters broad will hang the same bed? and the only term in it is 3 quarters of breadth.

The thing sought is the length or number of yards of cloth 3 quarters broad requisite to hang the bed, and the term in the supposition of the same kind is 30 yards, which therefore I place in the middle; the two remaining terms are extremes, and of the same kind, *viz.* both breadths.

It is easy to perceive that the answer must be greater than the middle term; for more yards of cloth 3 quarters broad will be necessary to hang a bed than of cloth 5 quarters broad, and the narrower the cloth is the more length will be required; wherefore the least extreme is

breadth. length. breadth.

qrs. yds. qrs.

3 : 30 :: 5

5

3)150(50 yards. *Ans.*

15

--

(0)

the divisor, which I place on the left hand, and set the greater extreme on the right.

And because the extreme found in the demand is the divisor, the proportion is inverse.

Quest. 3. If the penny-loaf weigh 8 ounces, when flour is sold at 2 s. or 24 d. *per* peck, what ought to be the weight of the penny-loaf when flour is sold at 18 d. *per* peck?

$$\begin{array}{rcl} d. & oz. & d. \\ 18 : 8 :: 24 & & 8 \end{array}$$

$$\begin{array}{r} 18 \overline{) 192} (10 \text{ oz.} \\ 18 \cdot \end{array}$$

Rem. 12 oz.

20

$$\begin{array}{r} 18 \overline{) 240} (13 \text{ dw.} \\ 18 \cdot \end{array}$$

60

54

Rem. 6 dw.

24

$$\begin{array}{r} 18 \overline{) 144} (8 \text{ gr.} \\ 144 \end{array}$$

(0)

Anf. 10 oz. 13 dw. 8 gr.

Quest. 4. If 1200 lb. weight be carried 36 miles for a sum of money, how many miles ought 1800 lb. weight to be carried for the same sum?

	<i>lb.</i>	<i>miles.</i>	<i>lb.</i>
Here the answer must be less than the middle term; because the heavier the load is, the fewer miles ought it to be carried for the same money; and so the greater extreme is the divisor, and placed on the left hand.	1800	: 36 ::	1200
			1200
	18	$\overline{) 432} \overline{) 00}$	(24 miles. <i>Anf.</i>
		36	
		72	
		72	
		(0)	

Quest.

Quest. 5. If L. 100 in 12 months gain a sum of interest, what principal will gain the same sum of interest in 8 months?

Here the answer must be greater than the middle term; because the fewer the months are, the more principal will be required to gain the same sum of interest; and therefore the least of the extremes is the divisor.

$$\begin{array}{rcl}
 m. & L. & m. \\
 8 & : 100 & :: 12 \\
 & & 100 \\
 & & \hline
 & 8) 1200 & \\
 & & \hline
 \text{Ans. } 150 & L. &
 \end{array}$$

Quest. 6. If 40 poles in length, and 4 in breadth, make an acre, what must be the length to make an acre when the breadth is 15 poles?

breadth. length. breadth.

$$15 : 40 :: 4$$

4

$$15) 160 \text{ (10 poles.)}$$

15

Rem. 10 poles.

33 half-feet in 1 pole.

— 2)

$$15) 330 \text{ (22 half feet.)}$$

30 —

— 11 feet.

30

30

—

(0)

poles. feet.

$$\text{Ans. } 10 \quad 11$$

Quest. 7. How much plush of 3 quarters wide will line a cloak that hath in it 4 yards of 7 quarters wide?

Y 2

Q.

Here the answer must be greater than the middle term; for the plush being narrower than the cloth of which the cloak is made, will require more length.

<i>Q. yd. Q.</i>
$3 : 4 :: 7$
$\underline{7}$
$3) 28 (9\frac{1}{3} \text{ yards. } \textit{Ans.}$
$\underline{27}$
(1)

Quest. 8. If 36 yards be a rood of mason-work, at 3 feet high, how many yards will make a rood at 9 feet high?

Feet. yards. feet.

$$9 : 36 :: 3$$

$$\underline{3}$$

$$9) 108$$

Ans. 12 yards.

M O R E E X A M P L E S.

Quest. 9. If a messenger makes a journey in 24 days when the day is 12 hours long, in how many days may he make out the same journey when the day is 16 hours long? *Ans.* in 18 days.

Quest. 10. If 360 men be in garrison, and have provisions only for 6 months, how many men must be turned out, that the same stock of provisions may last 9 months? *Ans.* 240 men are to be retained, and the rest, viz. 120, must be turned out.

Quest. 11. If I have 1200 lb. weight carried 36 miles for a sum of money, how many pound-weight shall I have carried 24 miles for the same sum? *Ans.* 1800 lb.

Quest. 12. If I borrow of a friend L. 64 for 8 months, what sum ought I to lend him for 12 months in order to requite the favour? *Ans.* L. 42 : 13 : 4.

Quest. 13. A fabric is reared in 8 months by 120 workmen; and another fabric, on the same plan, and of the same dimensions, is to be reared in 2 months: How many workmen must be employed? *Ans.* 480.

Quest.

Quest. 14. A regiment of soldiers, consisting of 1000 men, are to have new coats, each coat containing 3 yards 2 quarters of cloth that is 6 quarters wide, and they are to be lined with shalloon that is yard-wide : How many yards of cloth will the coats take ? and how many yards of shalloon will line them ? *Ans.* 3500 yards of cloth, and 5250 yards of shalloon.

Quest. 15. How many yards of paper that is 3 quarters wide will be sufficient to line a room that is 24 yards round and 4 yards high ? *Ans.* 128 yards.

Quest. 16. If I lend my friend L. 650 for 22 months, how long ought he to lend me L. 953 : 6 : 8 to requite my kindness ? *Ans.* 15 months.

Quest. 17. If 50 horses are maintained a year in corn for a certain sum, when oats are sold at 9 s. *per* boll, how many horses may be maintained a year in corn for the same sum, when oats are at 10 s. *per* boll ? *Ans.* 45 horses.

Quest. 18. If the penny-loaf weigh 10 oz. when wheat is sold at 12 s. *per* bushel, what ought to be the price of wheat when the penny-loaf weighs 8 oz. ? *Ans.* 15 s. *per* bushel.

Quest. 19. If 12 inches in length and 12 inches in breadth make a square foot, what length of a plank that is 9 inches broad will be equal to a square foot ? *Ans.* 16 inches.

Quest. 20. If 40 poles in length and 4 poles in breadth make an acre, what must be the breadth to make an acre when the length is only 32 poles ? *Ans.* 5 poles.

S E C T. II.

The Compound Rule of Three.

THE Compound Rule of Three, from five given numbers finds a sixth, or from seven given numbers finds an eighth, or from nine given numbers finds a tenth, or from eleven finds a twelfth, &c.

This rule easily and naturally admits of subdivisions, which, from the number of the terms given, may be denominated

denominated the rule of Five, the rule of Seven, the rule of Nine, the rule of Eleven, &c.

Questions in the compound rule of three are also resolved into two parts, *viz.* a supposition and a demand.

If five terms be given, three of these are always found in the supposition, and two in the demand; if seven terms be given, four of these are in the supposition, and three in the demand; if nine terms are given, five of these are in the supposition, and four in the demand; if eleven terms be given, six of these are in the supposition, and five in the demand, &c.

The supposition and demand being distinguished, proceed to state the question; that is, to put the terms in due order for operation, as the following rules direct.

R U L E I.

Place that term of the supposition which is of the same kind with the number sought, in the middle. The remaining terms are extremes, which must be classed into similar pairs, by making each pair consist of one term taken from the supposition, and another of the same kind taken from the demand.

R U L E II.

Out of each similar pair, joined with the middle term, form a simple question; and in each simple question, so formed, find the divisor; *viz.* consider from the nature of the question, whether the answer must be greater or less than the middle term; and if the answer must be greater, the least extreme is the divisor; but if the answer must be less than the middle term, the greatest extreme is the divisor.

R U L E III.

Place all the divisors on the left hand, and the other extremes on the right; then multiply the divisors, or extremes on the left, continually, for a divisor, and multiply the extremes on the right hand and the middle term, continually, for a dividend; and lastly, divide the

the dividend by the divisor; and the quot is the answer, of the same name with the middle term.

The answer to questions in the compound rule of three may also be had by working the simple questions separately, or by themselves, in the following manner, viz.

The middle term, with any one pair of similar extremes, make the first simple question, and the answer to this question must be made the middle term to the next similar pair of extremes; and the answer to this second question must, in like manner, be made the middle term to the following similar pair of extremes, &c.; and the answer to the last simple question is the number sought.

But the joint operation prescribed in Rule III. is the shorter as well as the easier method; for in working some of the simple questions, there may happen to be a remainder, and consequently the middle term of the next simple question will have some fractional part; which inconvenience is avoided by working jointly.

In every simple question, when the divisor is an extreme found in the supposition, the proportion is direct; but when the divisor is an extreme found in the demand, the proportion is inverse.

The three rules delivered above are indeed so calculated, as to make no difference betwixt direct and inverse, or so as to render that distinction needless, the left-hand extremes being all divisors; but yet, as questions consisting entirely of direct proportions are the plainest and easiest, it will be proper, in the first place, to exemplify the rules by questions of the direct kind, and afterwards introduce such as are inverse.

And as questions in the rule of five are by far more numerous, and occur much oftener, than questions in the rule of seven, nine, or eleven; I shall, first of all, adduce a set of questions in the rule of five, wherein both proportions are direct; then propose another set, wherein one or both proportions are inverse; and, lastly, give a few examples of the rules of seven, nine, and eleven.

I. The

I. *The Rule of Five direct.*

Quest. 1. If 14 horses eat 56 bushels of corn in 16 days, how many bushels will 20 horses eat in 24 days?

The supposition in this question is, If 14 horses eat 56 bushels in 16 days; and the three terms contained in it are, 14 horses, 56 bushels, and 16 days: The demand is, How many bushels will 20 horses eat in 24 days? and the two terms contained in it are 20 horses, and 24 days.

The number sought is bushels, and the term in the supposition of the same kind is 56 bushels; wherefore, according to Rule I. I place 56 bushels in the middle. The remaining four terms are extremes, which I class into similar pairs, by making each pair consist of one term taken from the supposition, and another of the same kind taken from the demand. Thus, 14 horses and 20 horses make one pair; again, 16 days and 24 days make another pair.

Out of the several similar pairs, joined with the middle term, I form so many simple questions, according to Rule II. *viz.* I say,

1. If 14 horses eat 56 bushels in a certain number of days, how many bushels will 20 horses eat in the same time?

2. If 16 days eat up, or consume, 56. or any other number of bushels, how many bushels will 24 days consume?

In the first simple question it is obvious, that the answer will be greater than the middle term; for 20 horses will eat more bushels than 14 horses will do in the same time; and so the least extreme, *viz.* 14, is the divisor; and because 14 is an extreme found in the supposition, the proportion is direct.

In the second simple question it is also plain, that the answer will be greater than the middle term; for 24 days will consume more bushels than 16 days; and consequently the least extreme, *viz.* 16, is the divisor; and
because

because 16 is an extreme found in the supposition, the proportion is direct.

According to Rule III. I place the divisors on the left hand, and the other extremes on the right, and both of them under one another; so that the two upper ones make a pair, or be of one kind, and the two lower ones make another pair, or be of one kind; and no matter which of the pairs be uppermost: then I multiply the divisors, or the extremes on the left hand, for a divisor; and again I multiply the extremes on the right, and the middle term, continually, for a dividend; and dividing the dividend by the divisor, the quot, or answer, comes out of the same name with the middle term, viz. 120 bushels.

The two simple questions into which the compound question is resolved, are stated, and wrought separately, as follows.

Joint operation.

<i>Horses.</i>	<i>bushels.</i>	<i>horses.</i>
If 14 :	56 ::	20
da. 16		24 da.
84		480
14		56
224		288
		240
	224) 26880 (120	
	224	
	448	
	448	
	(0)	

Ans. 120 bushels.

<i>H.</i>	<i>B.</i>	<i>H.</i>	<i>Days.</i>	<i>B.</i>	<i>Days.</i>
If 14 :	56 ::	20	If 16 :	80 ::	24
	20			80	

14) 1120 (80 B.

112

(0)

16) 1920 (120 B.

160

32

32

(0)

Ans. 120 bushels, as before.

Z

Quest.

Quest. 2. If 40 acres of grafs be cut down by 8 men in 7 days, how many acres shall be cut down by 24 men in 28 days ?

Joint operation.

<i>Men.</i>	<i>acres.</i>	<i>Men.</i>
If 8	: 40 ::	24
da. 7		28 da.

56

192

48

672

40

56) 26880 (480 Acres. *Anf.*

224

448

448

(0)

The same resolved into two simple questions, and wrought separately.

<i>M.</i>	<i>a.</i>	<i>M.</i>	<i>Days.</i>	<i>a.</i>	<i>Days.</i>
If 8	: 40 ::	24	If 7	: 120 ::	28
	40			120	

8) 960

7) 3360

120 acres.

Anf. 480 acres.

Quest. 3. If L. 100 in 12 months gain L. 5 interest, what will L. 75 gain in 9 months ?

Joint

Joint operation.

Pr. int. Pr.

L. L. L.

If 100 : 5 :: 75
months 12 9 months,

1200

675

5

L. s. d.

12|00) 33|75 (2 16 3 *Ans.*

24

975 *L. rem.*

20

195|00

12

75

72

3 *s. rem.*

12

36

36

(0)

The same question resolved into two simple ones, and wrought separately.

Z 2

Pr.

*Pr. int. Pr.**L. L. L.**Mo. L. s. Mo.*

If 100 : 5 :: 75

If 12 : 3—15 :: 9

5	20
—	—
L. s.	—
100) 375 (	3—15 75
20	9
—	—

1500	12) 675 (	20 L. s. d.
—	56 (2 16 3	Ans.
—	60 4	—
—	—	—

75 16 s.

72

Rem. 3 s.

12

36 (3 d.

36

(0)

Quest. 4. If a carrier receive 42 shillings for the carriage of 3 C. weight 150 miles, how much ought he to receive for the carriage of 7 C. 3 Q 14 lb. 50 miles?

Before the question is stated, it will be convenient to reduce one of the pairs as follows.

C. C. Q lb.

3 7—3—14

3 7

3 7

3 798

336 lb. 882 lb.

Joint

Joint operation.

$$\begin{array}{rcl} \text{lb.} & \text{s.} & \text{lb.} \\ \text{If } 336 : 42 :: 882 & & \\ \text{miles } 150 & & 50 \text{ miles.} \end{array}$$

$$\begin{array}{r} 1680 \\ 336 \\ \hline 50400 \end{array} \qquad \begin{array}{r} 44100 \\ 42 \\ \hline 882 \\ 1764 \end{array}$$

$$\begin{array}{r} 504|00 \quad 18522|00 \quad (36-9 \text{ Ans.} \\ 1512 \end{array}$$

$$\begin{array}{r} 3402 \\ 3024 \\ \hline \end{array}$$

Rem. 378 shill.
 12

$$\begin{array}{r} 4536(9 \text{ d.} \\ 4536 \\ \hline (0) \end{array}$$

The same question resolved into two simple ones, and wrought separately.

lb.

lb.	s.	lb.	m.	s.	d.	m.
If 336	:	42	::	882		
		42				
		1764				
		3528				
			s.	d.		
336	37044	(110--3	15	0	66	150(441
	336				60	
						36-9
	344				61	
	336				60	
Rem.	84	shill.			15	
	12				15	
	1008	(3 d.			(0)	
	1008					
	(0)				s.	d.
					Ans.	36-9

MORE EXAMPLES.

Quest. 5. If a regiment of 936 soldiers eat up 351 quarters of wheat in 168 days, how many quarters of wheat will 11,232 soldiers eat in 56 days at that rate? *Ans.* 1404 quarters.

Quest. 6. If 48 bushels of corn yield 576 bushels in 1 year, how much will 240 bushels yield in 6 years at that rate? *Ans.* 17,280 bushels.

Quest. 7. If 40 shillings be the wages of 8 men for 5 days, what will be the wages of 32 men for 24 days? *Ans.* 768 shillings, or L. 38, 8 s.

Quest. 8. If 8 cannons in 1 day spend 48 barrels of powder, how many barrels will 24 cannons spend in 12 days at that rate? *Ans.* 1728 barrels.

Quest. 9. An usurer receives L. 3 : 7 : 6 as the interest of L. 75 for 9 months: How much is that *per cent. per annum*? *Ans.* 6 *per cent.*

Quest. 10. If 18 roods of ditching be done by 3 men in

in 6 days, how many roods will be done by 8 men in 4 days at the same rate of working? *Ans.* 32 roods.

Quest. 11. If 4 students spend L. 19 in 3 months, how much will 8 students spend in 9 months? *Ans.* L. 114.

Quest. 12. If 600 seamen in 1 week eat 1500 lb. of beef, how much will serve 120 seamen 12 weeks? *Ans.* 3600 lb.

II. The Rule of Five inverse.

The questions that fall under this rule have commonly one of the proportions inverse, and the other direct, and sometimes the upper, and sometimes the lower, is the inverse proportion; and in some few questions both proportions are inverse. Now, though the three rules delivered above make no difference betwixt direct and inverse; yet, to bring the learner to some measure of acquaintance with this useful distinction, I shall, in stating the following questions, expose the same to view, by affixing an asterisk to the extremes of every inverse proportion.

Quest. 1. If 14 horses eat 56 bushels of corn in 16 days, in how many days will 20 horses eat 120 bushels at that rate?

In this question the supposition is, that 14 horses eat 56 bushels in 16 days; and the demand is, In how many days 20 horses will eat 120 bushels?

The number sought is days, and the term in the supposition of the same kind is 16 days; and accordingly I place 16 days in the middle. The remaining four terms are extremes; which I class into similar pairs, by making each pair consist of one term taken from the supposition, and another of the same kind taken from the demand. Thus, 14 horses and 20 horses make one pair; again, 56 bushels and 120 bushels make another pair.

Out of the similar pairs, joined with the middle term, I form so many simple questions; namely,

1. If 14 horses eat a certain number of bushels in 16 days, in how many days will 20 horses eat the same quantity?

2. If

2. If 56 bushels are eat up in 16 days, in how many days will 120 bushels be eat up by the same eaters?

In the first simple question it is plain, that the answer must be less than the middle term; for 20 horses will eat the same number of bushels in fewer days than 14 horses; and so the greatest extreme, *viz.* 20, is the divisor; and because 20 is an extreme found in the demand, the proportion is inverse.

In the second simple question it is also obvious, that the answer must be greater than the middle term; for 120 bushels will require more days to be eat up in, than 56 bushels; and therefore the least extreme, *viz.* 56, is the divisor; and because 56 is an extreme found in the supposition, the proportion is direct.

I now proceed to state the question, by placing the divisors on the left hand, and the other extremes on the right; then I multiply and divide, as directed in R. III. and the answer comes out of the same name with the middle term, *viz.* 24 days.

Joint operation.

Hors. day Hors.

* 20 : 16 :: 14 *

bush. 56 120 bush.

1120

1680

16

1008

168

--- days.

112|0)2688|0(.4 *Ans.*

224

448

448

(0)

The two simple questions into which the compound question is resolved, are stated and wrought separately, as follows.

Hors.

Horf. day. Horf.

* 20 : 16 :: 14 *

14

—

64

16

—..

2|0)22|4 (11 days.

24

9|6(4 hours.

—

16

60

—

96|0(48 min.

Busb. D. h. m.

56 : 11—4—48 :: 120

24

—

48

22

—

268

60

—

16128

120

— 6|0)

56(1935360(3456|0

108... —

— 24)576(24 days.

255

48

224

—

313

280

—

336

336

—

(0)

Quest. 2. If 40 acres of grafs be cut down by 8 men in 7 days, in how many days will 480 acres be cut down by 24 men?

A a

Joint.

Joint operation.

<i>Acrs.</i>	<i>days.</i>	<i>Acrs.</i>
40	: 7 ::	480
Men * 24		8 * Men.
960		3840
		7

		960)26880(28 days. <i>Anf.</i>
		192
		768
		768

		(0)

The same question resolved into two simple questions, and wrought separately.

<i>Acr. da.</i>	<i>Acr.</i>	<i>Men. da.</i>	<i>Men.</i>
40	: 7 ::	* 24	: 84 :: 8 *
	7		8
	-----		-----
40)3360(84 days.		24)672(28 days. <i>Anf.</i>	
32		48	
16		192	
16		192	
-----		-----	
(0)		(0)	

Quest. 3. If 100 l. principal, in 12 months, gain 5 l. interest, what principal will gain L. 2 : 16 : 3 interest in 9 months?

Joint

	L.	L.	s.	d.
<i>Joint operation.</i>	5	2	16	3
	240	20		
	<u> </u>	<u> </u>		
<i>Pr.</i>				
<i>Mon. L. Mon.</i>	1200d.	56		
* 9 : 100 :: 12 *		12		
Int. d. 1200	675 d. Int.	<u> </u>		
<u> </u>		675 d.		
10800	8100			
	100			
	<u> </u>			
108 00)8100 00(75 l.	<i>Ans.</i>			
756°				
<u> </u>				
540				
540				
<u> </u>				
(0)				

The same question resolved into two simple questions, and wrought separately.

<i>Mo. L. Mo.</i>	<i>D. L. s. d. D.</i>
* 9 : 100 :: 12 *	1200 : 133—6—8 :: 675
12	20
<u> </u>	<u> </u>
9)1200(133 l.	2666
3	12
20	<u> </u>
<u> </u>	32000
9)60(6 s.	675
54	<u> </u>
<u> </u>	1350
6	2025
12	<u> </u>
<u> </u>	12) 12
9)72(8 d.	12 00)216000 00(18000
72	12
<u> </u>	<u> </u>
(0)	2 0)150 0
	96
	<u> </u>
	96
	<u> </u>
	75 l. <i>Ans.</i>
	(000)
A a 2	<i>Quest.</i>

Quest. 4. If 12 inches of length, 12 of breadth, and 12 of thickness, make a solid foot, what length of a plank that is 6 inches broad and 4 inches thick will make a solid foot?

Joint operation.

Br. leng.	Br.	
* 6 : 12 :: 12 *		
Th. * 4	12 * Th.	
<u>24</u>	<u>144</u>	
	12	
	24)1728(72 inches. <i>Ans.</i>	
	<u>1680</u>	
	48	
	<u>48</u>	
	(0)	

The same question resolved into two simple questions, both inverse, and wrought separately.

Br. leng.	Br.
* 6 : 12 :: 12 *	
12	
<u>---</u>	
6)144	
<u>---</u>	
24 inches.	

Th. leng.	Th.
* 4 : 24 :: 12 *	
12	
<u>---</u>	
4)288	
<u>---</u>	
<i>Ans.</i> 72 inch. in leng.	

M O R E E X A M P L E S.

Quest. 5. If a carrier receive 42 shillings for the carriage of 3 C. weight 150 miles, how many miles ought he to carry C. 7 : 3 : 14 for 36s. 9d. *Ans.* 50 miles.

Quest. 6. If a regiment of 936 soldiers eat up 351 quarters of wheat in 168 days, how many soldiers will eat up 1404 quarters in 56 days at that rate? *Ans.* 11,232 soldiers.

Quest. 7. If 8 men in 5 days earn 40 shillings of wages, in how many days will 32 men earn 38l. 8s.? *Ans.* In 24 days.

Quest.

Quest. 8. If 8 cannons in 1 day spend 48 barrels of powder, in how many days will 24 cannons spend 1728 barrels at that rate? *Ans.* In 12 days.

Quest. 9. If 75 l. principal, in 9 months, gain L. 3 : 7 : 6. interest, in how many months will 100 l. principal gain 6 l. interest? *Ans.* In 12 months.

Quest. 10. If the penny-loaf weigh 12 ounces when wheat is sold at 3 s. 4 d. per bushel, what ought to be the weight of a loaf worth 9 d. when wheat is sold at 10 s. per bushel? *Ans.* 36 ounces.

Quest. 11. If one travels 300 miles in 10 days when the day is 12 hours long, in how many days may he travel 600 miles when the day is 16 hours long? *Ans.* In 15 days.

Quest. 12. If 48 pioneers, in 12 days, cast a trench 24 yards long, how many pioneers will cast a trench 168 yards long in 16 days? *Ans.* 252 pioneers.

III. *The Rule of Seven, Nine, Eleven, &c.*

Quest. 1. If 15 men eat 156 d. worth of bread in 6 days, when wheat is sold at 12 s. per bushel, in how many days will 30 men eat 520 d. worth of bread when wheat is at 10 s. per bushel?

This question belongs to the rule of seven, the number sought is days, and the term of the same kind in the supposition is 6 days, which I place in the middle. The remaining six terms are extremes, which I class into similar pairs, by taking one term of each pair out of the supposition, and another of the same kind out of the demand.

Out of the similar pairs, joined with the middle term, I form so many simple questions, in each of which I find the divisor by Rule II.; then I place the divisors on the left hand, and the other extremes on the right, as directed in Rule III. and multiply and divide, as follows.

Joint

*Joint operation.**Men. days. Men.*

* 30 : 6 :: 15 * s.
 * 10 : d. 156 520 d. : 12 *

4680

10

46800

30

75

7800

12

93600

6

468|00)5616|00(12 days. *Anf.*
 468

936

936

(0)

This compound question is resolved into three simple ones, as follows.

30 : 6 :: 15 : 3

156 : 3 :: 520 : 10

10 : 10 :: 12 : 12 days. *Anf.*

Quest. 2. If 18 men build a wall 40 feet long 3 feet thick and 16 feet high in 12 days, how many men will build a wall 360 feet long 8 feet thick and 10 feet high in 60 days?

This question belongs to the rule of nine, and is stated and wrought in the same manner with the preceding question in the rule of seven, as follows.

Joint

Joint operation.

Length. men. Length.

d. h. 40 : 18 :: 360 h. d.
* 60 : 16 : Th. 3 8 Th. : 10 : 12 *

120	2880
16	10
72	28800
12	12
1920	345600
60	18
115200	27648
	3456

1152|00)62208|00(54 men. *Ans.*

5760

4608

4608

(0)

This compound question is resolved into four simple ones, as follows.

40 : 18 :: 360 : 162

3 : 162 :: 8 : 432

16 : 432 :: 10 : 270

60 : 270 :: 12 : 54 men. *Ans.*

Quest. 3. If 12 men cast a ditch 30 feet long 6 deep and 3 broad in 15 days, when the day is 12 hours long; in how many days will 60 men cast a ditch 300 feet long 8 deep and 6 broad, when the day is but 8 hours long?

This question belongs to the rule of eleven, and is stated and wrought in the same manner with the preceding questions in the rules of seven and nine, as follows.

Joint

*Joint operation.**Men. days. Men.*

$$\begin{array}{lcl} \text{h. b. d.} & * 60 : 15 :: 12 * & \text{d. b. h.} \\ * 8 : 3 : 6 : 1. 30 & & 3001. : 8 : 6 : 12 * \end{array}$$

And $60 \times 30 \times 6 \times 3 \times 8 = 259200$ the divisor.And $12 \times 300 \times 8 \times 6 \times 12 \times 15 = 31104000$ the dividend.

$$\begin{array}{r} 2592|00) 311040|00 \text{ (120 days. Ans.} \\ 2592 \cdot \cdot \\ \hline \end{array}$$

$$\begin{array}{r} 5184 \\ 5184 \\ \hline \end{array}$$

(0)

The work may be contracted by omitting the equal extremes, 6 and 8, found both on the left hand and the right, thus.

 $60 \times 30 \times 3 = 5400$ the divisor.And $12 \times 300 \times 12 \times 15 = 648000$ the dividend.

$$54|00) 6480|00 \text{ (120 days. Ans.}$$

$$\begin{array}{r} 54 \cdot \cdot \\ \hline \end{array}$$

$$\begin{array}{r} 108 \\ 108 \\ \hline \end{array}$$

$$\begin{array}{r} 108 \\ \hline \end{array}$$

(0)

This compound question is resolved into five simple ones, as under.

$$60 : 15 :: 12 : 3$$

$$30 : 3 :: 300 : 30$$

$$6 : 30 :: 8 : 40$$

$$3 : 40 :: 6 : 80$$

$$8 : 80 :: 12 : 120 \text{ days. Ans.}$$

MORE EXAMPLES.

Quest. 4. If 18 roods of ditching be wrought by 3 men in 16 days, when the day is 15 hours long; how many

many roods will be wrought by 8 men in 4 days when the day is 9 hours long? *Ans.* $7\frac{1}{2}$ roods.

Quest. 5. If 12 men build a wall 30 feet long 6 feet high and 3 feet thick in 15 days, in how many days will 60 men build a wall 300 feet long 8 feet high and 6 feet thick? **Ans.** In 80 days.

Quest. 6. If 10 men, by working 6 hours a day, can, in 4 days, dig a cellar that is 24 feet long 16 wide and 12 deep; how many men, by working 8 hours a-day, will, in 2 days, dig a cellar that is 32 feet long 24 wide and 8 deep? *Ans.* 20 men.

Contractions in the Rule of Three.

1. If the first term is an aliquot part of the second or third, divide the said second or third term by the first, and the product of the quot and the other term is the answer. Thus, if $9 : 27 :: 42$, then divide 27 by 9, and the quot $3 \times 42 = 126$, the answer. Again, if $8 : 5 :: 64$, divide 64 by 8, and the quot $8 \times 5 = 40$, the answer.

E X A M P L E.

A, B, C, and D, have L. 100 to be divided among them, in such manner, that for every L. 3 A receives, B must have 5, C 7, and D 10; that is, their shares are to be as the numbers 3, 5, 7, 10: Required their shares?

Add the proportional numbers, by saying, $3+5+7+10=25$: then say,

If $25 : 100 ::$ $\left\{ \begin{array}{l} 3 \times 4 = 12 \text{ A's share.} \\ 5 \times 4 = 20 \text{ B's share.} \\ 7 \times 4 = 28 \text{ C's share.} \\ 10 \times 4 = 40 \text{ D's share.} \end{array} \right.$

100 proof.

Because 25 measures 100, I multiply the quot 4 into all the extremes on the right hand, and the products are the answers or shares.

2. If the first term be a multiple of the second or third,
 $B \quad b$ divide

divide the first term by the said second or third; and the remaining term, divided by the quot, gives the answer.

E X A M P L E I.

If 60 yards cost L. 20, what will 45 yards cost?

Here the first term 60 is a multiple of the second term 20, and $\frac{60}{20} = 3$, and $\frac{45}{3} = 15$ L. *Ans.*

E X A M P L E II.

If 18 yards cost L. 12, what will 6 yards cost?

Here the first term 18 is a multiple of the third term 6, and $\frac{18}{6} = 3$, and $\frac{12}{3} = 4$ L. *Ans.*

3. In compound questions the operation may frequently be rendered more simple, by placing the component parts of the divisor and dividend in a fractional form, rejecting such parts or factors of the numerator and denominator as happen to be equal, or cutting off an equal number of ciphers, or dividing by some common divisor.

E X A M P L E I.

If 35 ells of Vienna make 24 at Lyons, and 3 ells at Lyons make 5 at Antwerp, and 100 ells at Antwerp make 125 at Frankfort, how many ells at Frankfort make 42 at Vienna?

This question belongs to the rule of seven; and because the number sought is ells at Frankfort, therefore the given ells at Frankfort, *viz.* 125, is the middle term: the remaining terms are extremes; which I class into similar pairs, and state as follows.

Ant,

Ant. *Frank.* *Ant.*
 Vienna 100 : 125 :: 5 Vienna
 35 : Lyons 3 24 Lyons : 42

<u>300</u> <u>35</u> 10500	<u>120</u> <u>42</u> 5040 <u>125</u> 2520 1008 <u>504</u>
----------------------------------	---

105|00)6300|00(60 ells. *Ans.*
 630
 —
 (0)

The simple questions are,

100 : 125 :: 5 : $6\frac{1}{4}$
 3 : $6\frac{1}{4}$:: 24 : 50
 35 : 50 :: 42 : 60 ells at Frankfort. *Ans.*

But the question becomes more simple, by being stated and wrought in the fractional form, as follows.

$$\begin{aligned}
 & \frac{42 \times 24 \times 5 \times 125}{35 \times 3 \times 100} = \frac{42 \times 24 \times 5 \times 125}{3 \times 35 \times 100} \\
 & = \frac{42 \times 8 \times 1 \times 25}{1 \times 7 \times 20} = \frac{8400}{140} = \frac{840}{14} = 60 \text{ ells at Frankfort. } \textit{Ans.}
 \end{aligned}$$

E X A M P L E II

If 100 lb. of Venice weigh 70 lb. of Lyons, and 120 lb. of Lyons weigh 100 lb. of Roan, and 80 lb. of Roan weigh 100 lb. of Tolouse, and 100 pound of Tolouse weigh 74 lb. of Geneva, how many pounds of Geneva will 100 lb. of Venice weigh?

B b 2

This

This question belongs to the rule of nine; and because pounds of Geneva is the number sought, the given pounds of Geneva, *viz.* 74, must be the middle term: the remaining terms are extremes; which may be classed into similar pairs, and stated as follows.

	<i>Tol. Gen.</i>	<i>Tol.</i>	
Ver. Ly.	100 : 74 ::	100	Ly. Ven.
100 : 120 : Roan	80	100	Roan : 70 : 100
	8000	10000	
	120	70	
	960000	700000	
	100	100	
	96000000	70000000	
		74	
	96000000) 5180000000 (53 $\frac{22}{8}$ lb. of Ge-		
	480		neva. <i>Ans.</i>
		380	
		288	
		(92)	

But the question becomes more simple, and is wrought with greater ease and advantage, by being stated in the fractional form, as follows.

$$\frac{100 \times 70 \times 100 \times 100 \times 74}{100 \times 120 \times 80 \times 100} = \frac{70 \times 100 \times 74}{120 \times 80} =$$

$$\frac{7 \times 10 \times 74}{12 \times 8} = \frac{5180}{96} = 53\frac{22}{8} \text{ lb. of Geneva. } \textit{Ans.}$$

I shall now conclude, by observing, that every compound question, whether in the rule of five, seven, nine, or eleven, &c. properly and strictly speaking, consists but of three given terms. For the first term, or divisor, is

is to be considered as one compound term made up, or produced, by the continual multiplication of the extremes on the left hand, as so many component parts. In like manner, the third term is to be considered as one compound term, made up by the continual multiplication of the extremes on the right, as component parts. Suppose the question to be,

If L. 100 in 12 months gain L. 5 interest, what will L. 75 gain in 9 months?

Here it is obvious, that it is neither the L. 100 principal, nor the 12 months of time, taken separately, that gains the L. 5 interest, but both contribute their share; that is, they conspire, as joint causes, to produce one effect; and therefore their product, *viz.* the first term, is to be considered as the cause producing the effect; that is, the first term, *viz.* 100×12 , causeth, produceth, or gains, L. 5 of interest. And in like manner, the product of the extremes on the right hand, or the third term, *viz.* 75×9 , is to be esteemed the cause that produceth a similar effect; that is, gains a like sum of interest, namely, the fourth term, or answer. In reference to this way of considering the first and third terms, the question might be stated as under.

$$\text{If } 100 \times 12 : 5 :: 75 \times 9$$

C H A P VIII.

F E L L O W S H I P.

FELLOWSHIP, called also *Company*, or *Partnership*, is, when two or more persons join their stocks, and trade together, dividing the gain or loss proportionally among the partners.

Fellowship is either without or with time, called also *single* or *double*.

I. Fellowship without time.

Questions in fellowship without time are wrought by the following proportion.

As the total stock

To the total gain or loss,

So each man's particular stock

To his share of the gain or loss.

Quest 1. A and B make a joint stock : A puts in 12 l. and B 8 l. ; they gain 5 l. : What is each man's share ?

	L.	Stock. gain	Stock.
A's stock	12	A. If 20 : 5 :: 12	
B's stock	8		5
Total stock	20		2 0)6 0

	Stock. gain.	Stock.	A's gain	3 l.
B. If 20 : 5 :: 8				
	8		L.	
	—	A's gain	3	
	2 0)4 0	B's gain	2	
	—			
B's gain	2 l.	Total gain	5	proof.

Quest 2. A, B, and C, make a joint stock : A puts in 78 l. B 117 l. and C 234 l. ; they gain 265 l. : What is each man's share ?

L.

	L.	Stock.	gain.	Stock.
Stock of	{ A 78	A. If 429 : 265 :: 78		
	{ B 117		78	
	{ C 234			
			2120	
Total stock	429		1855	

429)20670(481.

1716

3510

3432

78

20

429)1560(3 s.

1287

273

12

429)3276(7 d.

3003

273

4

429)1092(2 f.

858

(234)

Stock.

Stock. gain. Stock.
 B. If 429 : 265 :: 117

117

1855

265

265

429)31005(72 l.

3003.

975

858

117

20

429)2340(5 s.

2145

195

12

429)2340(5 d.

2145

195

4

429)780(1 f.

429

(351)

Stock. gain. Stock.
 C. If 429 : 265 :: 234

234

1060

795

530

429)62010(144 l.

429..

1911

1716

1950

1716

234

20

429)4680(10 s.

429

390

12

429)4680(10 d.

429

390

4

429)1560(3 f.

1287

(273)

Anf.

		L.	s.	d.	f.	Rem.
Ans. Gain of	{ A	48—	3—	7—	2—	234
	{ B	72—	5—	5—	1—	351
	{ C	144—	10—	10—	3—	273
Proof		265—	0—	0—	0—	858

Note 1. When in any question there happen to be remainders, they must be reduced equally low, so as to be all of one name; and then their sum will be either equal to the divisor, or exactly double, triple, &c. of it: and accordingly 1, 2, 3, &c. carried from the sum of the remainders, and added to the particular gains, will make up the total gain; or the divisor will always divide the sum of the remainders exactly, and the quot added to the particular gains will give the total gain.

Note 2. When the partners have equal shares of stock or capital, their shares of gain, loss, or neat proceeds, is found readily by dividing the total gain, loss, &c. by the number of partners, thus.

Suppose A, B, and C, were equally concerned in a voyage to Virginia, and that the neat proceeds turned out to 575 l. each partner's share will be L. 191 : 13 : 4, cast up as under.

$$\begin{array}{r} \text{L.} \quad \text{s.} \quad \text{d.} \\ 3)575(191 \quad 13 \quad 4 \end{array}$$

Note 3. When the fractions denoting the partners shares of stock or capital have all the same denominator, their shares of gain, loss, &c. is quickly found, by dividing the total gain, &c. by the denominators, and multiplying each quot by its respective numerator, thus.

Suppose A, B, C, and D, buy a ship for a sum of money, whereof A, B, and C, pay $\frac{1}{2}$ each, and D $\frac{3}{4}$ or $\frac{1}{2}$; and that her freight on a voyage amounts to 260 l. the partners shares are found as follows.

C.c

6)260

6)260

L. 43 : 6 : 8 to A.

43 : 6 : 8 to B.

43 : 6 : 8 to C.

130 : 0 : 0 to D.

Proof 260

Note 4. The operation may sometimes be shortened or facilitated by first finding the gain, loss, or neat proceeds, *per cent.* or *per pound*, and then working by the rules of practice, thus.

Suppose A, B, and C, join in an adventure to Barbadoes, which, with all charges, amounted to 2000 l.; whereof A paid 400 l. B 600 l. and D 1000 l.; they have returns in goods, which being disposed of, the neat proceeds amounted to L. 4333 : 6 : 8 : What should each partner get ?

If 2000 : 4333— 6—8 :: 100

20 : 4333— 6—8 :: 1

10 : 216|6—13—4 :: 1

20

—

13|3

12

—

4|0

216 13 4 *per cent.*

4

400 = 866 13 4

200 = 433 6 8

600 = 1300

1000 = 2166 13 4

L. s. d.

A gets 866 13 4

B gets 1300

C gets 2166 13 4

Proof 4333 6 8

Again,

Again, Suppose A, B, and C, join in an adventure to Jamaica, and ship off, on their joint credit, goods to the value of 3000 l.; and, to complete the cargo, the partners, from their own warehouses, ship off such goods as they had proper for the voyage, viz. A goods to the value of 100 l. B 200 l. C 300 l.; the neat proceeds of their returns in sugar and rum amounted to 6930 l.: What share of this belongs to each partner?

$$\begin{aligned} 3600 &: 6930 :: 1 \\ 360 &: 693 :: 1 \\ 120 &: 231 :: 1 \\ 40 &: 77 :: 1 : \frac{77}{40} \text{ per l.} \end{aligned}$$

					Shares.
					L. s.
A's stock	1100	$\times 77 =$	84700,	$\div 40 =$	2117 10
B's stock	1200	$\times 77 =$	92400,	$\div 40 =$	2310
C's stock	1300	$\times 77 =$	100100,	$\div 40 =$	2502 10

Proof 6930

M O R E E X A M P L E S.

Quest. 3. A, B, and C, make a joint stock: A puts in 460 l. B 510 l. and C 480 l.; they gain 340 l.: What is each partner's share?

	L.	s.	d.	f.	Rem.
<i>Ans.</i> Gain of { A	107	17	2	3	85
{ B	119	11	8	2	110
{ C	112	11	0	1	95
Proof.	340	00	0	0	290

Quest. 4. Three persons trade together: A puts in 230 l. B 529 l. and C 344 l. 10 s.; they gain 520 l.: What is that to each?

	L.	s.	d.	Rem.
<i>Ans.</i> Gain of { A	108	7	7 $\frac{3}{4}$	271
{ B	249	5	7	844
{ C	162	6	9	1092
C c 2				

Quest.

Quest. 5. A, B, and C, make a joint stock of 3256l.: whereof A puts in 1026l. B 985l. and C the rest; by misfortune they lose 2000l.: What part of that must each bear?

	L.	s.	d.	Rem.
<i>Ans.</i> { A	630	— 4	— 5	— 928
B	605	— 0	— 8 $\frac{3}{4}$	— 1240
C	764	— 14	— 10	— 1088

Quest. 6. Four partners, A, B, C, and D, built a ship, which cost 1730l.; and the freight for her first voyage amounted to 370l.; of which A's share was 74l. B's 111l. C's 148l. and D's 37l.: What was each partner's stock?

	L.
<i>Ans.</i> { A's stock was	346
B's	— — — 519
C's	— — — 692
D's	— — — 173

Total stock 1730 Proof.

Quest. 7. A, B, and C, in company, gain 36l.: A put in 20l. B 30l. C a sum unknown; C took up 16l. of the gain: What did A and B gain? and what did C put in?

	L.
<i>Ans.</i> { A's gain	8
B's gain	12
C's stock	40

Quest. 8. A, B, C, and D, in partnership, had a joint stock of 600l. and gained a certain sum; of which A, B, and C, took up 60l.; B, C, and D, 90; A, C, and D, 80; A, B, and D, 70: What was the stock and gain of each partner?

Stock.

		Stock.	Gain.
		L.	L.
Ans.	{ A's	60	10
	{ B's	120	20
	{ C's	180	30
	{ D's	240	40

II. Fellowship with time.

In fellowship with time, the gain or loss is divided among the partners, both in proportion to the stocks themselves, and also in proportion to the times of their continuance in company: for the same stock continued a double time, procures a double share of gain; and continued a triple time, procures a triple share of gain; that is, the shares of gain or loss are as the products of the several stocks multiplied into their respective times: and accordingly questions belonging to this rule are wrought by the following proportion.

As the sum of the products of the several stocks into their respective times

To the total gain or loss,

So the product of each man's stock into his time

To his share of the gain or loss.

Quest. 1. A put into company 40 l. for 3 months, B 75 l. for 4 months; they gain 70 l.: What share must each man have?

A $40 \times 3 = 120$, third term for A's share.

B $75 \times 4 = 300$, third term for B's share.

420, first term.

L. A. If $420 : 70 :: 120$ 120 42 0)840 0(20 l. 84 (0)	L. B. If $420 : 70 :: 300$ 300 42 0)2100 0(50 l. 210 (0)
---	---

▲'s

	L.
A's gain	20
B's gain	50
	—

Total gain 70 proof.

Quest. 2. A put into company 560 l. for 8 months, B 279 l. for 10 months, and C 735 l. for 6 months; they gained 1000 l.: What share of the gain must each have?

A $560 \times 8 = 4480$, third term for A's share.

B $279 \times 10 = 2790$, third term for B's share.

C $735 \times 6 = 4410$, third term for C's share.

11680, first term.

	L.		L.	s.	d.	f.	Rem.
A If	11680	:	1000	::	4480	:	383—11—2—3— 208
B If	11680	:	1000	::	2790	:	238—17—4—3— 80
C If	11680	:	1000	::	4410	:	377—11—4—1— 880
							—
Proof	1000	—	00	—	0	—	0— 1168

M O R E E X A M P L E S.

Quest. 3. A, B, and C, hire a pasture for 24 l.: A puts in 40 cows for 4 months, B 30 cows for 2 months, and C 36 cows for 5 months: What share of the rent must each pay?

		L.	s.
<i>Ans.</i> {	A's share of rent	9	12
	B's share - -	3	12
	C's share - -	10	16
		—	—

Proof 24

Quest. 4. A, B, and C, agreeing to trade together, A puts in 529 l. for 4 months, B 329 l. for 7 months, and C 900 l. for 2 months; they gain 540 l.: What is each man's share?

Ans.

	L.	s.	d.	Rem.
<i>Ans.</i> { A	183—	14—	8 —	2304
B	199—	19—	5 —	1222
C	156—	5—	10 $\frac{3}{4}$ —	2583

Quest. 5. A and B enter into partnership for a year: accordingly A put in the first of January 50 l.; but B could not put any money in till the first of May: What sum must B then put in to be intitled to an equal share of the gain at the year's end? *Ans.* 75 l.

Quest. 6. A, B, and C agree to trade in company for 12 months: A put in at first 364 l. and at the end of 4 months he put in 40 l. more; B put in at first 408 l. and at the end of 7 months he took out 86 l.; C put in at first 148 l. and at the end of 3 months he put in 86 l. more, and at the end of other 5 months he put in 100 l. more; at the year's end their gain was 1436 l.: What is each man's share?

	L.	s.	d.	Rem.
<i>Ans.</i> { A's gain	556—	3—	6 —	6192
B's gain	529—	16—	9 —	5496
C's gain	349—	19—	8 —	416

Note. Questions in double fellowship are proper exercise for the learner, but seldom occur in real business; differences in point of time being usually adjusted by an interest-accompt.

CHAP. IX.

VULGAR FRACTIONS.

A FRACTION is a part or parts of an unit, or of any integer or whole; and is expressed by two numbers, one above and the other below a line drawn between them; as, $\frac{1}{4}$.

The number under the line shews into how many parts the unit or integer is divided; and is called the *denominator*, because it gives name to the fraction: the number above the line shews or tells how many of these parts the fraction contains; and is therefore called the *numerator*.

In the fraction $\frac{3}{4}$ l. a pound Sterling is the unit, integer,

teger, or whole; and the denominator 4 shews that the pound is broken or divided into four equal parts, viz. 4 crowns; and the numerator 3 shews that the fraction contains three of these parts, that is, three crowns; and so the value of this fraction is fifteen shillings.

C O R O L L A R Y I.

Hence it follows, 1. When the numerator of a fraction is less than the denominator, the value of such a fraction is less than unity, or the integer. 2. When the numerator is equal to the denominator, the value of the fraction is exactly an unit or integer. 3. When the numerator is greater than the denominator, the value of the fraction is more than an unit; and so often as the denominator is contained in the numerator, so many units or wholes are contained in the fraction. If, therefore, the numerator of a fraction be divided by the denominator, the quot will be a number of units or integers, and the remainder so many parts.

The numerator of a fraction is to be considered as a dividend, and the denominator as a divisor; and the fraction itself may be taken to denote the quotient.

C O R O L L A R Y II.

From this view of a fraction, it is evident, that if the numerator and denominator of a fraction be either both multiplied or both divided by the same number, the products or quotients will retain the same proportion to one another; and consequently the new fraction thence arising will be of the same value with the given one. Thus the numerator and denominator of the fraction $\frac{2}{4}$ multiplied by 2 produces $\frac{4}{8}$, and divided by 2 quots $\frac{1}{2}$, both which fractions are of the same value with $\frac{2}{4}$.

Fractions having 10, 100, 1000, or 1 with any number of ciphers annexed to it, for a denominator, are called *decimal fractions*; and fractions having any other denominator are called *vulgar fractions*.

But the doctrine of vulgar fractions, or the rules therein laid down for reducing, adding, subtracting, multiplying, and dividing fractions, are equally applicable

to

to fractions of every kind; to the decimal as well as the vulgar; and therefore, in what follows, the decimal and vulgar fractions are promiscuously used in the exemplification of the rules.

Decimal fractions, indeed, may be added, subtracted, &c. and have their operations conducted, not only by the rules assigned in the doctrine of vulgar fractions, which are of a general or universal nature, and extend alike to fractions of every sort; but also by a more easy and simple method, peculiar to themselves, which vulgar fractions do not admit of; and this may be called *the doctrine of decimals*; the explication whereof is reserved to some subsequent part of this treatise. In the mean time, we proceed to describe the other classes or kinds into which fractions in general are divided.

1. A proper fraction is that whose numerator is less than its denominator, and consequently is in value less than unity; as $\frac{2}{3}$.

2. An improper fraction is that whose numerator is equal to or greater than its denominator; and consequently is in value equal to or greater than an unit; as $\frac{4}{4}$, $\frac{7}{5}$.

3. A simple fraction is that which has but one numerator, and one denominator: and may be either proper or improper; as $\frac{5}{8}$ or $\frac{2}{7}$.

4. A compound fraction is made up of two or more simple fractions, coupled together with the particle *of*, and is a fraction of a fraction; as $\frac{2}{3}$ of $\frac{3}{4}$, or $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.

5. A mixt number consists of an integer, and a fraction joined with it; as $7\frac{3}{4}$.

Because in most cases fractions can neither be added nor subtracted, till they be reduced, we begin with reduction.

Reduction of Vulgar Fractions.

P R O B. I.

To reduce an improper fraction to an integer, or mixt number.

D d

R U L E.

R U L E.

Divide the numerator by the denominator, the quotient gives integers; and the remainder, if there be any, placed over the divisor or denominator, gives the fraction to be annexed.

E X A M P L E S.

1. $\frac{525}{8} = 85$ integers, there being no remainder.

2. $\frac{437}{8} = 54\frac{5}{8}$, the remainder being 5.

3. $\frac{1382}{14} = 98\frac{10}{14}$, the remainder being 10.

4. $\frac{23576}{136} = 173\frac{48}{136}$, the remainder being 48.

The reason of the rule is evident from Corollary I. For the denominator of every fraction is one whole; and if the numerator be divided by the denominator, the quotient will be units, integers, or wholes, and the remainder so many parts.

P R O B. II.

To reduce a mixt number to an improper fraction.

R U L E.

Multiply the integer by the denominator; to the product add the numerator: the sum is the numerator of the improper fraction; and the denominator is the same as before.

E X A M P L E S.

1. $54\frac{5}{8} = \frac{437}{8}$; for $54 \times 8 = 432$

$$\begin{array}{r} + 5 \\ \hline \end{array}$$

Numerator 437

2. $98\frac{10}{14} = \frac{1382}{14}$; for $98 \times 14 = 1372$

$$\begin{array}{r} + 10 \\ \hline \end{array}$$

Numerator 1382

$$3. \ 173 \frac{48}{136} = \frac{23576}{136}, \text{ for } 173 \times 136 = 23528$$

$$\begin{array}{r} 23528 \\ + 48 \\ \hline \end{array}$$

Numerator 23576

The reason of this rule needs not be assigned, both the rule and the examples being the reverse of the foregoing.

P R O B. III.

To reduce a whole number to a fraction of a given denominator.

R U L E.

Multiply the whole number by the given denominator; and place the product by way of numerator over the given denominator.

E X A M P L E S.

1. Reduce 9 to a fraction whose denominator is 5.

$$9 \times 5 = 45; \text{ so the fraction is } \frac{45}{5}.$$

2. Reduce 36 to a fraction whose denominator is 4.

$$36 \times 4 = 144; \text{ so the fraction is } \frac{144}{4}.$$

3. Reduce 8 to a fraction whose denominator is 1.

$$8 \times 1 = 8; \text{ so the fraction is } \frac{8}{1}.$$

It is easy to perceive, that the fractions of this problem will all be improper; and from Ex. 3. we may learn, that any whole number may be reduced to the form of a fraction, by making unity the denominator.

The reason of the rule appears by reversing the operation; for if the numerator be divided by the denominator, it will quot the integer, or whole number.

P R O B. IV.

To reduce a compound fraction to a simple one.

R U L E.

Multiply the numerators continually for the numerator

D d 2

tor

tor of the simple fraction; and multiply the denominators continually for its denominator.

EXAMPLES.

Ex. 1.

$$\frac{2}{3} \text{ of } \frac{4}{5} = \frac{8}{15}.$$

Ex. 2.

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{3}{4} = \frac{6}{24}.$$

An operation may sometimes be shortened by connecting the numerators and denominators with the sign of multiplication, and then rejecting such of the factors above and below as happen to be the same. Thus, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5} = \frac{2 \times 3 \times 4}{3 \times 4 \times 5}$, and rejecting 3 and 4, both above and below, the fraction is quickly reduced to $\frac{2}{5}$.

The reason of the rule will appear by reviewing Ex. 1.; in which $\frac{4}{5}$, by corollary II. is equal to $\frac{4 \times 3}{5 \times 3} = \frac{12}{15}$; and one third of $\frac{12}{15}$ is $\frac{4}{5}$; consequently two thirds is $\frac{8}{15} = \frac{2 \times 4}{3 \times 5}$.

COROLLARY.

From this problem may be deduced a method of reducing a fraction of a lesser denomination to a fraction of a greater denomination: namely,

Form a compound fraction, by comparing the given fraction with the superior denominations; and then reduce the compound fraction to a simple one.

EXAMPLES.

1. What fraction of a pound Sterling is $\frac{3}{4}$ of a penny?

$$\frac{3}{4} \text{ d. is } \frac{3}{4} \text{ of } \frac{1}{12} \text{ of } \frac{1}{20} \text{ L.} = \frac{3}{800} \text{ L.}$$

2. What fraction of a C. is $\frac{7}{8}$ of a pound?

$$\frac{7}{8} \text{ lb. is } \frac{7}{8} \text{ of } \frac{1}{16} \text{ of } \frac{1}{4} \text{ C.} = \frac{7}{512} \text{ C.}$$

3. What fraction of a pound Troy is $\frac{4}{5}$ of a penny-weight?

$$\frac{4}{5} \text{ dw. is } \frac{4}{5} \text{ of } \frac{1}{20} \text{ of } \frac{1}{12} \text{ lb.} = \frac{4}{1200} \text{ lb. Troy.}$$

PROB.

P R O B. V.

To reduce a fraction of a greater denomination to a fraction of a lesser denomination.

R U L E.

Multiply the numerator of the given fraction, as in reduction of integers descending; and the product is the numerator, to be placed over the denominator of the given fraction.

E X A M P L E S.

1. What fraction of a shilling is $\frac{3}{4}$ of a pound?

Here, as in reduction descending, I multiply the numerator 3 by 20, because 20 shillings make a pound; as under.

$$\begin{array}{l} L. \\ \frac{3 \times 20}{4} = \frac{60}{4} \text{ shilling.} \end{array}$$

2. What fraction of a penny is $\frac{4}{5}$ L.?

$$\begin{array}{l} L. \\ \frac{4 \times 20 \times 12}{5} = \frac{960}{5} d. \end{array}$$

3. What fraction of a farthing is $\frac{2}{3}$ L.?

$$\begin{array}{l} L. \\ \frac{2 \times 20 \times 12 \times 4}{3} = \frac{1920}{3} f. \end{array}$$

4. What fraction of a pound is $\frac{7}{8}$ C.?

$$\begin{array}{l} C. \\ \frac{7 \times 4 \times 28}{8} = \frac{784}{8} lb. \end{array}$$

5. What fraction of a grain is $\frac{5}{12}$ lb. Troy?

$$\begin{array}{l} lb. \\ \frac{5 \times 12 \times 20 \times 24}{12} = \frac{28800}{12} gr. \end{array}$$

6. What fraction of an inch is $\frac{1}{2}$ yard?

Yard.

$$\frac{1 \times 3 \times 12}{2} = \frac{36}{2} \text{ inch.}$$

Yard

$$\text{Or } \frac{1 \times 36}{2} = \frac{36}{2} \text{ inch.}$$

The reason of this rule will appear by observing, that every fraction may be considered in two views. Thus, $\frac{3}{4}$ may either be considered as expressing three fourths of one unit, or as denoting the fourth part of three units. Now, if the unit be a pound Sterling, the fraction, in the latter view, will denote the fourth part of three pounds; and by reducing the numerator L. 3 to shillings, we have $\frac{60}{4}$ s.; and again reducing 60 shillings to pence, we have $\frac{720}{4}$ d. equal to $\frac{60}{4}$ s. or to $\frac{3}{4}$ L.

P R O B. VI.

To find the value of a fraction.

R U L E.

Reduce the numerator to the next inferior denomination; divide by the denominator; and the quot, if nothing remain, is the value complete.

If there be any remainder, it is the numerator of a fraction whose denominator is the divisor. This fraction may either be annexed to the quotient, or reduced to value, if there be any lower denomination.

EX.

EXAMPLES.

1. What is the value of $\frac{3}{4}$ L. ?

Here I consider $\frac{3}{4}$ L. as expressing the fourth part of three pounds Sterling; so I reduce L. 3, the numerator, to shillings, and divide by the denominator 4; and as nothing remains, the quot, viz. 15 shillings, is the value complete.

$$\begin{array}{r} 3 \\ 20 \\ \hline 4)60(15\text{ s.} \\ 4 \\ \hline 20 \\ 20 \\ \hline (0) \end{array}$$

$$\begin{array}{l} \text{L.} \quad \text{s.} \\ \frac{3}{4} = 15 \end{array}$$

2. What is the value of $\frac{2}{16}$ L. ?

Here there is a remainder of 4, which makes $\frac{4}{16}$ of a shilling; this I reduce to value, viz. I multiply 4 by 12, the pence in a shilling, and divide the product by the denominator 16; and the quot gives 3 d.

$$\begin{array}{r} 9 \\ 20 \\ \hline 16)180(11\text{ s.} \\ 16 \\ \hline 20 \\ 16 \\ \hline 4\text{ rem.} \\ 12 \\ \hline 16)48(3\text{ d.} \\ 48 \\ \hline (0) \end{array}$$

$$\begin{array}{l} \text{L.} \quad \text{s.} \quad \text{d.} \\ \frac{2}{16} = 11 - 3 \end{array}$$

3. What

3. What is the value of $\frac{16}{21}$ C.?

$$\begin{array}{r} 16 \\ 21 \overline{) 64} \quad (3 \text{ Q.} \\ \underline{63} \\ 1 \end{array}$$

Here the last remainder 7 makes $\frac{7}{21}$ of a pound; which I annex to the quotient.

$$\begin{array}{r} 21 \overline{) 28} \quad (1 \frac{7}{21} \text{ lb.} \\ \underline{21} \\ 7 \end{array}$$

(7)

$$\begin{array}{l} \text{C.} \quad \text{Q.} \quad \text{lb.} \\ \frac{16}{21} = 3 - 1 \frac{7}{21} \end{array}$$

MORE EXAMPLES.

4. What is the value of $\frac{27}{9}$ L. Sterling?

Ans. 18 s. 7 d. $1 \frac{2}{3}$ f.

5. What is the value of $\frac{53}{94}$ of a tun?

Ans. 11 C. 1 Q. $2 \frac{22}{94}$ lb.

6. What is the value of $\frac{6}{18}$ lb. Troy?

Ans. 4 oz. 10 dw.

7. What is the value of $\frac{41}{50}$ of a year?

Ans. 299 days 7 hours 12 minutes.

The reason of this rule is the same with that in the preceding problem. It is by the practice of this problem that remainders in the rule of three are reduced to value.

P R O B. VII.

To reduce a fraction to its lowest terms.

R U L E.

Divide both numerator and denominator by their greatest

greatest common divisor; the two quotients make the new fraction.

The greatest common divisor of the numerator and denominator of a fraction is found by the following rule.

Divide the greater of these two numbers by the lesser; and again divide the divisor by the remainder; and so on, continually, till 0 remains. The last divisor is their greatest common divisor.

EXAMPLES.

1. Reduce $\frac{784}{952}$ to its lowest terms.

First I find the greatest common divisor of the numerator and denominator, as follows.

$$784)952(1$$

$$\underline{784}$$

$$168)784(4$$

$$\underline{672}$$

$$112)168(1$$

$$\underline{112}$$

Greatest common divisor 56(112(2

$$\underline{112}$$

$$(0)$$

I now proceed to reduce the given fraction to its lowest terms, by dividing both numerator and denominator by 56, the greatest common divisor.

$$56)784(14 \text{ new num.}$$

$$\underline{56}$$

$$224$$

$$\underline{224}$$

$$(0)$$

$$56)952(17 \text{ new denom.}$$

$$\underline{56}$$

$$392$$

$$\underline{392}$$

$$(0)$$

$$\text{So } \frac{784}{952} = \frac{14}{17}$$

E e

2. Reduce

2. Reduce $\frac{846}{468}$ to its lowest terms.

First find the greatest common divisor, as under.

$$468 \overline{)846}(1$$

$$\underline{468}$$

$$378 \overline{)468}(1$$

$$\underline{378}$$

$$90 \overline{)378}(4$$

$$\underline{360}$$

Greatest common divisor $18 \overline{)90}(5$

$$\underline{90}$$

$$(0)$$

Now reduce the given fraction to its lowest terms.

$18 \overline{)846}(47$ new num. $18 \overline{)468}(26$ new denom.

$$\underline{72}$$

$$\underline{126}$$

$$\underline{126}$$

$$(0)$$

$$\underline{36}$$

$$\underline{108}$$

$$\underline{108}$$

$$(0)$$

$$\text{So } \frac{846}{468} = \frac{47}{26} = 1 \frac{21}{26}$$

3. Reduce

3. Reduce $\frac{147}{294}$ to its lowest terms.

$$\begin{array}{r} 147 \overline{) 323} 2 \\ 294 \\ \hline \end{array}$$

$$\begin{array}{r} 29 \overline{) 147} 5 \\ 145 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \overline{) 29} 14 \\ 2 \\ \hline \end{array}$$

$$2$$

$$9$$

$$8$$

Greatest common divisor 1) 2(2

$$2$$

$$(0)$$

The greatest common divisor being unity, the given fraction is already in its lowest terms.

MORE EXAMPLES.

4. What is $\frac{208}{84}$ in its lowest terms? *Ans.* $\frac{52}{21}$.

5. What is $\frac{2548}{12}$ in its lowest terms? *Ans.* $\frac{7}{8}$.

6. What is $\frac{2728}{96}$ in its lowest terms? *Ans.* $\frac{11}{2}$.

This way of abbreviating or reducing a fraction to its lowest terms, by finding the greatest common divisor, being tedious, you may use the following method.

Divide the numerator and denominator of the given fraction by any number that will divide both without any remainder, and you have your fraction in lower terms. In like manner, reduce this new fraction to lower terms still, and so on continually, till no common divisor, except unity, is found, and you have your fraction in its lowest terms.

In order to discover what number will divide both numerator and denominator without a remainder, observe the following practical rules or directions.

1. If the units, or right hand figure, of both nume-

E c 2

rator

rator and denominator are even numbers, you may always halve them, or divide by 2.

$$\text{Ex. 1. } \overset{2}{\cancel{784}} \mid \overset{2}{\cancel{392}} \mid \overset{2}{\cancel{196}} \mid \overset{7}{\cancel{28}} \mid \overset{1}{\cancel{4}}$$

In the above example I proceed by halving, till the new fraction is $\frac{98}{119}$, where 9, the unit of the denominator, is an odd number, so 2 cannot measure both; consequently neither can 4, 6, or 8, the multiples of 2. By trial I find 3 cannot measure both, and so neither can 6 or 9, the multiples of 3. Then I try 7, and find it measures both. So the given fraction is reduced to $\frac{14}{17}$, and is in its lowest terms, unity being the only number that measures both numerator and denominator.

The divisors in the above example may be considered as component parts of the greatest common divisor, and accordingly the product arising from their continual multiplication, *viz.* $2 \times 2 \times 2 \times 7 = 56$, is the greatest common divisor.

$$\text{Ex. 2. } \overset{2}{\cancel{192}} \mid \overset{2}{\cancel{96}} \mid \overset{2}{\cancel{48}} \mid \overset{2}{\cancel{24}} \mid \overset{2}{\cancel{12}} \mid \overset{2}{\cancel{6}} \mid \overset{3}{\cancel{3}} \mid \overset{1}{\cancel{1}}$$

2. If the right-hand figure of both numerator and denominator be 5, or if the right-hand figure of the one be 5 and of the other 0, you may in either case divide by 5.

$$\text{Ex. 1. } \overset{5}{\cancel{375}} \mid \overset{5}{\cancel{75}} \mid \overset{5}{\cancel{15}} \mid \overset{3}{\cancel{9}} \mid \overset{1}{\cancel{3}}$$

$$\text{Ex. 2. } \overset{5}{\cancel{525}} \mid \overset{5}{\cancel{105}} \mid \overset{3}{\cancel{21}} \mid \overset{7}{\cancel{7}}$$

$$\text{Ex. 3. } \overset{5}{\cancel{140}} \mid \overset{7}{\cancel{28}} \mid \overset{1}{\cancel{4}}$$

3. If there are ciphers on the right hand of both numerator and denominator, cut off an equal number of ciphers from both; for to cut off one cipher is the same

same thing as to divide by 10, and to cut off two is the same as to divide by 100, &c.

$$\text{Ex. 1. } \begin{array}{c} 10) \\ 180 \\ \hline 18 \end{array} \bigg| \begin{array}{c} 2) \\ 18 \\ \hline 9 \end{array} \bigg| \begin{array}{c} 3) \\ 9 \\ \hline 3 \end{array} \bigg| \frac{3}{4}$$

$$\text{Ex. 2. } \begin{array}{c} 100) \\ 21000 \\ \hline 210 \end{array} \bigg| \begin{array}{c} 5) \\ 210 \\ \hline 42 \end{array} \bigg| \begin{array}{c} 7) \\ 42 \\ \hline 6 \end{array} \bigg| \begin{array}{c} 3) \\ 6 \\ \hline 2 \end{array} \bigg| \frac{2}{3}$$

$$\text{Ex. 3. } \frac{3000}{5000} = \frac{3}{5}$$

The reason of this rule has been already assigned in Corollary II. *viz.* When the numerator and denominator of a fraction are both divided by the same number, the new fraction formed by the quotients is of the same value with the given one.

The rule for finding the greatest common divisor may be thus demonstrated: If any number measure both the remainder of a division, and also the divisor, it will likewise measure the dividend; for the dividend consists of two parts, whereof one is a multiple of the divisor, produced by multiplying the quotient into the divisor; and the other part is the remainder itself: now, since the supposed number measures the remainder, and also measures the divisor, and all its multiples, it will measure both parts of the dividend; that is, it will measure the dividend.

Thus, in Ex. 1. since every number measures itself, the remainder 56, in the third division, will measure itself; but it also measures the divisor 112; and therefore will also measure the dividend 168; that is, it measures the remainder and the divisor of the second division; and consequently measures the dividend 784; that is, it measures both the remainder and the divisor of the first division; and therefore will also measure the dividend 952, and at the same time is the highest or greatest number that will do so.

P R O B. VIII.

To reduce fractions of different denominators to a common denominator.

R U L E

R U L E

Multiply the denominators continually for the common denominator; and multiply each numerator into all the denominators, except its own, for the several numerators.

E X A M P L E S.

1. Reduce $\frac{3}{4}$ and $\frac{4}{5}$ to a common denominator.

$4 \times 5 = 20$, the common denominator.

$3 \times 5 = 15$, the first numerator.

$4 \times 4 = 16$, the second numerator.

So the new fractions are $\frac{15}{20}$ and $\frac{16}{20}$.

2. Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{8}$, to a common denominator.

$3 \times 6 \times 8 = 144$, the common denominator.

$2 \times 6 \times 8 = 96$, the first numerator.

$5 \times 3 \times 8 = 120$, the second numerator.

$7 \times 3 \times 6 = 126$, the third numerator.

The new fractions are $\frac{96}{144}$, $\frac{120}{144}$, and $\frac{126}{144}$.

3. Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{6}{7}$, $\frac{9}{10}$, to a common denominator.

$2 \times 5 \times 7 \times 10 = 700$, the common denominator.

$1 \times 5 \times 7 \times 10 = 350$, the first numerator.

$3 \times 2 \times 7 \times 10 = 420$, the second numerator.

$6 \times 2 \times 5 \times 10 = 600$, the third numerator.

$9 \times 2 \times 5 \times 7 = 630$, the fourth numerator.

New fractions, $\frac{350}{700}$, $\frac{420}{700}$, $\frac{600}{700}$, $\frac{630}{700}$.

M O R E E X A M P L E S.

$$4. \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}, = \frac{720}{960}, \frac{768}{960}, \frac{800}{960}, \frac{840}{960}.$$

$$5. \frac{1}{3}, \frac{4}{5}, \frac{5}{7}, \frac{2}{8}, = \frac{315}{280}, \frac{420}{280}, \frac{675}{280}, \frac{378}{280}.$$

$$6. \frac{1}{4}, \frac{5}{7}, \frac{9}{16}, \frac{1}{2}, = \frac{140}{560}, \frac{400}{560}, \frac{504}{560}, \frac{280}{560}.$$

When the denominator of one fraction happens to be an aliquot part of the denominator of another fraction, the former may be reduced to the same denominator with the latter, by multiplying both its numerator and denominator by the number which denotes how often the lesser denominator is contained in the greater.

$$\text{Thus, } \frac{2}{3} + \frac{5}{12} = \frac{8}{12} + \frac{5}{12}.$$

Here

Here 3 is contained in 12 four times; so I multiply both 2 and 3 by 4, and I have $\frac{8}{12} = \frac{2}{3}$.

Again, $\frac{1}{2} + \frac{3}{4} + \frac{5}{8} = \frac{4}{8} + \frac{6}{8} + \frac{5}{8}$.

Sometimes, too, the fraction that has the greater denominator may, in like manner, be reduced to the same denominator with that which has the lesser, by division.

Thus, $\frac{3}{9} + \frac{2}{3} = \frac{1}{3} + \frac{2}{3}$.

And $\frac{16}{48} + \frac{9}{36} + \frac{7}{12} = \frac{4}{12} + \frac{3}{12} + \frac{7}{12}$.

The reason of the above rule for reducing fractions to a common denominator is evident from Corollary II.; for both numerator and denominator of every fraction are multiplied by the same number, or by the same numbers.

After fractions are reduced to a common denominator, they may frequently be reduced to lower terms, by dividing all the numerators, and also the common denominator, by any divisor that leaves no remainder, or by cutting off an equal number of ciphers from both.

The lower fractions are reduced, the more simple and easy will any operation be; if, therefore, it be required to reduce fractions to the lowest or least common denominator possible, it may be done as follows.

The least multiple of all the given denominators is the least common denominator.

Divide this common denominator severally by the denominators of the given fractions; and multiply each numerator respectively by the quot arising from its own denominator; and the products will be the numerators.

The least multiple of two or more numbers is found thus.

Place the given numbers in a line; then divide two of them, or as many of them as you can, by 2, or 3, or any small divisor that leaves no remainder; place the quots, and the numbers not divided, in a line below; again, divide the numbers in this line, either by the former, or by some other divisor, placing the quots, and the numbers not divided, in a line below; proceed by dividing, in the same manner, till the last quot of every number

number be unity ; then multiply the divisors into one another continually, and their product is the least multiple required.

EXAMPLE I.

Required the least multiple of 24 and 36 ?

Divisors 6 24. 36.

2	4.	6.
2	2.	3.
3	1.	3.
		1.

$6 \times 2 \times 2 \times 3 = 72$, the least multiple.

EXAMPLE II.

Required the least multiple of 24, 36, 54 ?

Divisors 6 24. 36. 54.

3	4.	6.	9.
2	4.	2.	3.
2	2.	1.	3.
3	1.		3.
			1.

$6 \times 3 \times 2 \times 2 \times 3 = 216$, the least multiple.

EXAMPLE III.

Required the least multiple of 2, 4, 6, 10, 12, 15 ?

Divisors 2 2. 4. 6. 10. 12. 15.

2	1.	2.	3.	5.	6.	15.
3		1.	3.	5.	3.	15.
5			1.	5.	1.	5.
				1.		1.

$2 \times 2 \times 3 \times 5 = 60$, the least multiple.

The

The reason of the operation is obvious. For the continual product of the divisors used in dividing any of the given numbers is equal to the said given number; and if this product be multiplied by the remaining divisor or divisors, the product will be a multiple of the given number. Thus, in Ex. 1. the divisors used in dividing 24 are 6, 2, 2, and $6 \times 2 \times 2 = 24$, the given number; and if we multiply this by 3, the remaining divisor, we shall have $6 \times 2 \times 2 \times 3 = 72$, a multiple of 24. In like manner, the divisors used in dividing 36 are 6, 2, 3, and $6 \times 2 \times 3 = 36$, the given number; and if we multiply this by 2, the remaining divisor, we shall have, as before, $6 \times 2 \times 2 \times 3 = 72$, a multiple of 36, which, at the same time, is the least common multiple of 24 and 36.

If any of the smaller given numbers be an aliquot part of some of the higher given numbers, you may, if you please, neglect the aliquot parts. Thus, in Ex. 3. you may neglect 2, 4, and 6, as being aliquot parts of 12; and by dividing 10, 12, 15, you will have 60 for the least common multiple, as before.

I now proceed to the exemplification of the above rule.

Ex. 1. Reduce $\frac{5}{12}$ and $\frac{7}{18}$ to their least common denominator.

Divisors	6	12.	18.	$6 \times 2 \times 3 = 36$, the least common denominator.
	2	2.	3.	
	3	1.	3.	
			1.	

12)36($3 \times 5 = 15$ the first numerator.

18)36($2 \times 7 = 14$ the second numerator.

The new fractions are $\frac{15}{36}$ and $\frac{14}{36}$.

Ex. 2 Reduce $\frac{5}{8}$, $\frac{7}{12}$, $\frac{4}{9}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{4}$, to their least common denominator.

F f

Div.

Div. 2 | 8. 12. 9. 3. 6. 4.

24. 6. 9 3. 3. 2.

22. 3. 9. 3. 3. 1.

3. 1. 3. 9. 3. 3.

3 1. 3. 1. 1.

1.

$2 \times 2 \times 2 \times 3 \times 3 = 72$,
the common denominator.

8)72(9 $\times 5 = 45$ first numerator.
 12)72(6 $\times 7 = 42$ second numerator.
 9)72(8 $\times 4 = 32$ third numerator.
 3)72(24 $\times 2 = 48$ fourth numerator.
 6)72(12 $\times 5 = 60$ fifth numerator.
 4)72(18 $\times 1 = 18$ sixth numerator.

New fractions $\frac{45}{72}, \frac{42}{72}, \frac{32}{72}, \frac{48}{72}, \frac{60}{72}, \frac{18}{72}$.

The method of finding the new numerators may be thus accounted for. The value of a fraction is not altered by multiplying both its numerator and denominator by the same number; but when the common denominator is divided by the denominator of the given fraction, the quot gives the number into which the denominator of the given fraction is multiplied; and consequently its numerator must be multiplied by the same. Thus, in the last example, the common denominator, 72, divided by 8, the denominator of the given fraction $\frac{5}{8}$, quotes 9; and so the new fraction is formed thus,

$$\frac{5 \times 9}{8 \times 9} = \frac{45}{72}; \text{ in like manner, the new fraction } \frac{7}{12} \text{ is}$$

formed thus, $\frac{7 \times 6}{12 \times 6} = \frac{42}{72}$ cc.

Addition of Vulgar Fractions.

Rule I. If the given fractions have all the same denominator, add the numerators, and place the sum over the denominator.

Ex. 1. What is the sum of $\frac{2}{7} + \frac{3}{7}$? *Ans.* $\frac{5}{7}$.

Ex.

Ex. 2. What is the sum of $\frac{3}{15} + \frac{6}{15}$? *Ans.* $\frac{9}{15} = \frac{3}{5}$, by Prob. VII.

Ex. 3. What is the sum of $\frac{3}{8} + \frac{5}{8} + \frac{7}{8}$? *Ans.* $\frac{15}{8} = 1 \frac{7}{8}$, by Prob. I.

Rule II. If the given fractions have different denominators, reduce them to a common denominator, by Prob. VIII. then add the numerators, and place the sum over the common denominator.

Ex. 1. What is the sum of $\frac{2}{8} + \frac{3}{8}$?

$$\frac{2}{8} + \frac{3}{8} = \frac{16}{40} + \frac{15}{40}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{16}{40} + \frac{15}{40} = \frac{31}{40}.$$

Ex. 2. What is the sum of $\frac{1}{3} + \frac{3}{4} + \frac{5}{6}$?

$$\frac{1}{3} + \frac{3}{4} + \frac{5}{6} = \frac{24}{72} + \frac{54}{72} + \frac{60}{72}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{24}{72} + \frac{54}{72} + \frac{60}{72} = \frac{138}{72}.$$

$$\text{and } \frac{138}{72} = 1 \frac{66}{72}, \text{ by Prob. I.} = 1 \frac{11}{12}, \text{ by Prob. VII.}$$

The operation becomes more simple, if the given fractions be reduced to the least common denominator. The former example wrought in this manner follows.

What is the sum of $\frac{1}{3} + \frac{3}{4} + \frac{5}{6}$?

$$\frac{1}{3} + \frac{3}{4} + \frac{5}{6} = \frac{4}{12} + \frac{9}{12} + \frac{10}{12} = \frac{23}{12} = 1 \frac{11}{12}, \text{ by Prob. I.}$$

Rule III. If mixt numbers be given, or if mixt numbers and fractions be given, reduce the mixt numbers to improper fractions, by Prob. II.; then reduce the fractions to a common denominator, by Prob. VIII. and add the numerators.

Ex. 1. What is the sum of $7 \frac{3}{4} + 5 \frac{2}{3}$?

$$7 \frac{3}{4} + 5 \frac{2}{3} = \frac{31}{4} + \frac{17}{3}, \text{ by Prob. II.}$$

$$\text{and } \frac{31}{4} + \frac{17}{3} = \frac{93}{12} + \frac{68}{12}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{93}{12} + \frac{68}{12} = \frac{161}{12} = 13 \frac{5}{12}, \text{ by Prob. I.}$$

F f 2

Ex.

Ex. 2. What is the sum of $6\frac{2}{3} + \frac{4}{5}$?

$$6\frac{2}{3} = \frac{20}{3}, \text{ by Prob. II.}$$

$$\text{and } \frac{20}{3} + \frac{4}{5} = \frac{100}{15} + \frac{12}{15}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{100}{15} + \frac{12}{15} = \frac{112}{15} = 7\frac{7}{15}, \text{ by Prob. I.}$$

When mixt numbers, or mixt numbers and fractions, are given, you may, with greater expedition, work by the following rule, *viz.* reduce only the fractions to a common denominator, and add the sum of the fractions to the integers. The above two examples wrought in this manner follow.

Ex. 1. What is the sum of $7\frac{3}{4} + 5\frac{2}{3}$?

$$\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}.$$

$$\text{and } 7 + 5 + 1\frac{5}{12} = 13\frac{5}{12}.$$

Ex. 2. What is the sum of $6\frac{2}{3} + \frac{4}{5}$?

$$\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1\frac{7}{15}.$$

$$\text{and } 6 + 1\frac{7}{15} = 7\frac{7}{15}.$$

Rule IV. If any, or all of the given fractions, be compound, first reduce the compound fractions to simple ones, by Prob. IV.; then reduce the simple fractions to a common denominator, by Prob. VIII. and add the numerators.

Ex. 1. What is the sum of $\frac{2}{3}$ of $\frac{4}{5} + \frac{3}{4}$?

$$\frac{2}{3} \text{ of } \frac{4}{5} = \frac{8}{15}, \text{ by Prob. IV.}$$

$$\text{and } \frac{8}{15} + \frac{3}{4} = \frac{32}{60} + \frac{45}{60}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{32}{60} + \frac{45}{60} = \frac{77}{60} = 1\frac{17}{60}, \text{ by Prob. I.}$$

Ex. 2. What is the sum of $\frac{1}{2}$ of $\frac{2}{3} + \frac{3}{5}$ of $\frac{1}{4}$?

$$\frac{1}{2} \text{ of } \frac{2}{3} + \frac{3}{5} \text{ of } \frac{1}{4} = \frac{2}{6} + \frac{3}{20}, \text{ by Prob. IV.}$$

$$\text{and } \frac{2}{6} + \frac{3}{20} = \frac{40}{120} + \frac{18}{120}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{40}{120} + \frac{18}{120} = \frac{58}{120} = \frac{29}{60}, \text{ by Prob. VII.}$$

Rule V. If the given fractions be of different denominations, first reduce them to the same denomination, by Cor. of Prob. IV. or by Prob. V.; then reduce the fractions,

fractions, now of one denomination, to a common denominator, by Prob. VIII. and add the numerators; or reduce each of the given fractions separately to value, by Prob. VI. and then add their values.

EXAMPLE.

What is the sum of $\frac{3}{4}$ s. and $\frac{7}{8}$ l.?

METHOD I.

$$\frac{3}{4} \text{ s.} = \frac{3}{4} \text{ of } \frac{1}{20} \text{ l.} = \frac{3}{80} \text{ l. by Cor. Prob. IV.}$$

$$\text{and } \frac{3}{80} + \frac{7}{8} = \frac{24}{640} + \frac{560}{640}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{24}{640} + \frac{560}{640} = \frac{584}{640} \text{ l.} = 18 \text{ s. 3 d. by Prob. VI.}$$

METHOD II.

$$\frac{7}{8} \text{ l.} = \frac{7 \times 20}{8} \text{ s.} = \frac{140}{8} \text{ s. by Prob. V.}$$

$$\text{and } \frac{3}{4} + \frac{140}{8} = \frac{6}{8} + \frac{140}{8}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{6}{8} + \frac{140}{8} = \frac{146}{8} \text{ s.} = 18 \text{ s. 3 d. by Prob. I. and VI.}$$

METHOD III.

$$\begin{array}{rcl} \frac{3}{4} \text{ s.} = \frac{3 \times 12}{4} \text{ d.} = \frac{36}{4} \text{ d.} = & \text{d.} & \\ & 9 & \\ \frac{7}{8} \text{ l.} = \frac{7 \times 20}{8} \text{ s.} = \frac{140}{8} \text{ s.} = & \text{s.} & \\ & 17-6 & \\ & 18-3 & \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{by Prob. VI.}$$

MORE EXAMPLES.

1. What is the sum of $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$? *Ans.* $1 \frac{7}{8}$.
2. What is the sum of $\frac{7}{8} + \frac{11}{12} + \frac{17}{18}$? *Ans.* $2 \frac{53}{72}$.
3. What is the sum of $13 \frac{3}{4} + 24 \frac{5}{8}$? *Ans.* $38 \frac{11}{8}$,
or $38 \frac{7}{2}$.
4. What is the sum of $\frac{3}{4}$ of $\frac{3}{8} + \frac{9}{10}$ of $\frac{6}{7} + \frac{5}{8}$? *Ans.*
 $1 \frac{237}{280}$.
5. What

5. What is the sum of $\frac{3}{4}l. + \frac{3}{4}s. + \frac{3}{4}d.$? *Ans.*
 $\frac{253}{20}l.$ in value 15 s. 9 d. 3 f.

The reason of the above rules appears from Axiom XI. because none but similar or like things can be added. Thus, third parts cannot be added to fourth parts, for the sum would neither be third parts nor fourth parts; third parts can only be added to third parts, and fourth parts only to fourth parts; and consequently fractions to be added must have all the same denominator. In like manner, the fraction of a pound cannot be added to the fraction of a shilling, or of a penny, being unlike things; and therefore fractions to be added must be all of the same denomination.

Subtraction of Vulgar Fractions.

Rule I. If the given fractions have the same denominator, subtract the lesser numerator from the greater, and place the remainder over the denominator.

Ex. 1. From $\frac{5}{7}$ subtract $\frac{3}{7}$.

$$\frac{5}{7} - \frac{3}{7} = \frac{2}{7}.$$

Ex. 2. From $\frac{10}{13}$ subtract $\frac{4}{13}$.

$$\frac{10}{13} - \frac{4}{13} = \frac{6}{13}.$$

Rule II. If the given fractions have different denominators, reduce them to a common denominator, by Prob. VIII.; then subtract the lesser numerator from the greater, and place the remainder over the common denominator.

Ex. 1. From $\frac{3}{4}$ subtract $\frac{2}{3}$.

$$\frac{3}{4}, \frac{2}{3} = \frac{9}{12}, \frac{8}{12}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{9}{12} - \frac{8}{12} = \frac{1}{12}.$$

Ex. 2. From $\frac{9}{10}$ subtract $\frac{7}{8}$.

$$\frac{9}{10}, \frac{7}{8} = \frac{72}{80}, \frac{70}{80}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{72}{80} - \frac{70}{80} = \frac{2}{80} = \frac{1}{40}, \text{ by Prob. VII.}$$

Rule

Rule III. If it be required to subtract one mixt number from another, or to subtract a fraction from a mixt number, reduce the mixt numbers to improper fractions, by Prob. II.; then reduce the fractions to a common denominator, by Prob. VIII. and subtract the one numerator from the other.

Ex. 1. From $7\frac{3}{4}$ subtract $5\frac{1}{2}$.

$7\frac{3}{4}, 5\frac{1}{2} = \frac{31}{4}, \frac{11}{2}$, by Prob. II.

and $\frac{31}{4}, \frac{11}{2} = \frac{62}{8}, \frac{44}{8}$, by Prob. VIII.

and $\frac{62}{8} - \frac{44}{8} = \frac{18}{8} = 2\frac{2}{8} = 2\frac{1}{4}$, by Prob. I. and VII.

Ex. 2. From $8\frac{2}{3}$ subtract $\frac{4}{5}$.

$8\frac{2}{3} = \frac{26}{3}$, by Prob. II.

and $\frac{26}{3}, \frac{4}{5} = \frac{130}{15}, \frac{12}{15}$, by Prob. VIII.

and $\frac{130}{15} - \frac{12}{15} = \frac{118}{15} = 7\frac{13}{15}$, by Prob. I.

In subtracting one mixt number from another, or in subtracting a fraction from a mixt number, you may, with greater ease and expedition, proceed thus, *viz.* Reduce only the fractions to a common denominator, and then work by one or other of the two rules following.

1. If the numerator of the fraction to be subtracted be less than the numerator of the fraction from which you are to subtract, then subtract the lesser numerator from the greater, place the remainder over the common denominator, and to this fraction prefix the difference of the integers of the two mixt numbers; or the integral part of the mixt number when a fraction is subtracted.

Ex. 1. From $7\frac{3}{4}$ subtract $5\frac{1}{2}$.

$\frac{3}{4}, \frac{1}{2} = \frac{6}{8}, \frac{4}{8}$ by Prob. VIII.

and $\frac{6}{8} - \frac{4}{8} = \frac{2}{8} = \frac{1}{4}$ by Prob. VII.

and $7 - 5 = 2$.

So the difference of the two mixt numbers is $2\frac{1}{4}$.

Ex.

Ex. 2. From $9\frac{3}{4}$ subtract $\frac{2}{3}$.

$\frac{3}{4}, \frac{2}{3} = \frac{9}{12}, \frac{8}{12}$ by Prob. VIII.

and $\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$; and to this fraction I prefix the integral part of the mixt number, and the difference, or remainder, is $9\frac{1}{12}$.

Note 1. In subtracting a fraction from an unit, you have only to subtract the numerator from the denominator; and the remainder, placed over the denominator, is the difference, or answer. Thus, $1 - \frac{3}{5} = \frac{2}{5}$: for $1 = \frac{5}{5}$, and $\frac{5}{5} - \frac{3}{5} = \frac{2}{5}$.

2. If the numerator of the fraction to be subtracted be greater than the numerator of the fraction from which you are to subtract; in this case subtract the greater fraction from an unit borrowed; that is, subtract the greater numerator from the common denominator, add the remainder to the numerator of the lesser fraction, and place the sum over the common denominator, for the fractional part of the answer; or, which is the same in effect, subtract the greater numerator from the sum of the numerator and denominator of the lesser fraction, and place the remainder over the common denominator for the fractional part of the answer: then, for the unit thus borrowed, add 1 to the integral part of the lesser mixt number; subtract this sum from the integral part of the greater mixt number; and prefix the difference to the fractional part of the answer. But in subtracting a fraction from a mixt number, for the unit borrowed, take 1 from the integral part of the mixt number, and prefix the remainder to the fractional part of the answer.

Ex. 1. From $8\frac{1}{2}$ subtract $5\frac{2}{3}$.

$\frac{1}{2}, \frac{2}{3} = \frac{3}{6}, \frac{4}{6}$, by Prob. VIII.

Here having reduced the fractions to a common denominator, I say, $\frac{4}{6}$ from $\frac{3}{6}$ I cannot; wherefore I subtract $\frac{4}{6}$ from an unit borrowed, viz. I subtract the numerator 4 from the common denominator 6; and add the remainder 2 to the numerator of the lesser fraction

3; and the sum 5, placed over the common denominator, gives $\frac{5}{8}$ for the fractional part of the answer. Or, which is the same in effect, I subtract 4 from $3 + 6 = 9$, and there remains $\frac{5}{8}$ for the fractional part of the answer, as before. Then, for the unit borrowed, I say, 1 borrowed and 5 make 6; which, subtracted from 8, leaves 2 of a remainder; and this prefixed to the fractional part, gives $2\frac{5}{8}$ for the difference, remainder, or answer.

Ex. 2. From $9\frac{2}{3}$ subtract $\frac{4}{3}$.

$\frac{2}{3}, \frac{4}{3} = \frac{10}{3}, \frac{12}{3}$, by Prob. VIII.

Here having reduced the fractions to a common denominator, I say, $\frac{12}{3}$ from $\frac{10}{3}$ I cannot; but, borrowing an unit, I subtract the numerator 12 from the common denominator 15, and 3 remains; which 3, added to the lesser numerator 10, gives $\frac{13}{3}$ for the fractional part of the answer. Or, I subtract 12 from $10 + 15 = 25$, and 13 remains; which 13, placed over the common denominator, gives $\frac{13}{3}$ for the fractional part, as before. Then, for the unit borrowed, I take 1 from the integral part of the mixt number; and the remainder, prefixed to the fractional part, gives $8\frac{13}{3}$ for the difference, or answer.

Rule IV. If it be required to subtract a mixt number, or a fraction, from an integer, first subtract the fraction from an unit borrowed; that is, subtract the numerator from the denominator, and place the remainder, as a numerator, over the denominator, for the fractional part of the answer: then, for the unit borrowed, add 1 to the integral part of the mixt number; subtract the sum from the given integer; and prefix the remainder to the fractional part of the answer. But when a fraction is subtracted from an integer, for the unit borrowed, take 1 from the given integer, and prefix the remainder to the fractional part of the answer.

Ex. 1. From 14 subtract $7\frac{2}{5}$.

Here I say, $5 - 3 = 2$; so $\frac{2}{5}$ is the fractional part of
G g the

the answer: then I say, 1 borrowed and 7 make 8, and 8 subtracted from 14 leaves 6; which I prefix to the fractional part: so the difference or answer is $6\frac{2}{5}$.

Ex. 2. From 12 subtract $\frac{3}{7}$.

Here I say, $7 - 3 = 4$; so $\frac{4}{7}$ is the fractional part: then I say, 1 borrowed from 12, and 11 remains: so $11\frac{4}{7}$ is the difference, or answer.

Note 2. When an integer is given to be subtracted from a mixt number, you have only to subtract the given integer from the integral part of the mixt number; and to the remainder annex the fractional part. Thus, $9\frac{2}{3} - 5 = 4\frac{2}{3}$.

Rule V. If one or both of the given fractions be compound, first reduce the compound fractions to simple ones, by Prob. IV.; then reduce the simple fractions to a common denominator, by Prob. VIII.; and subtract the one numerator from the other.

Ex. 1. From $\frac{4}{5}$ subtract $\frac{2}{3}$ of $\frac{3}{4}$.

$$\frac{2}{3} \text{ of } \frac{3}{4} = \frac{6}{12}, \text{ by Prob. IV.}$$

$$\text{and } \frac{4}{5}, \frac{6}{12} = \frac{48}{60}, \frac{30}{60}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{48}{60} - \frac{30}{60} = \frac{18}{60} = \frac{3}{10}, \text{ by Prob. VII.}$$

Ex. 2. From $\frac{2}{3}$ of $\frac{1}{2}$ subtract $\frac{3}{5}$ of $\frac{1}{4}$.

$$\frac{2}{3} \text{ of } \frac{1}{2}, \frac{3}{5} \text{ of } \frac{1}{4} = \frac{2}{6}, \frac{3}{20}, \text{ by Prob. IV.}$$

$$\text{and } \frac{2}{6}, \frac{3}{20} = \frac{40}{120}, \frac{18}{120}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{40}{120} - \frac{18}{120} = \frac{22}{120} = \frac{11}{60}, \text{ by Prob. VII.}$$

Rule VI. When the given fractions are of different denominations, first reduce them to the same denomination, by Cor. of Prob. IV. or by Prob. V; then reduce the fractions, now of one denomination, to a common denominator, by Prob. VIII.; and subtract the one numerator from the other. Or, reduce each of the given fractions, separately, to value, by Prob. VI.; and subtract the one value from the other.

EX.

EXAMPLE.

From $\frac{3}{4}$ l. subtract $\frac{2}{3}$ s.

METHOD I.

$\frac{2}{3}$ s. = $\frac{2}{3}$ of $\frac{1}{20}$ l. = $\frac{2}{60}$ l. by Cor. Prob. IV.

and $\frac{3}{4}$, $\frac{2}{60} = \frac{180}{240}$, $\frac{8}{240}$, by Prob. VIII.

and $\frac{180}{240} - \frac{8}{240} = \frac{172}{240}$ l. = 14 s. 4 d. by Prob. VI.

METHOD II.

$\frac{3}{4}$ l. = $\frac{3 \times 20}{4}$ s. = $\frac{60}{4}$ s. by Prob. V.

and $\frac{60}{4}$, $\frac{2}{3} = \frac{180}{12}$, $\frac{8}{12}$, by Prob. VIII.

and $\frac{180}{12} - \frac{8}{12} = \frac{172}{12}$ s. = 14 s. 4 d. by Prob. I. and VI.

METHOD III.

$$\left. \begin{array}{l} \frac{3}{4} \text{ l.} = \frac{3 \times 20}{4} \text{ s.} = \frac{60}{4} \text{ s.} = 15 - 0 \\ \frac{2}{3} \text{ s.} = \frac{2 \times 12}{3} \text{ d.} = \frac{24}{3} \text{ d.} = 8 \end{array} \right\} \begin{array}{l} \text{s.} \quad \text{d.} \\ 15 - 0 \\ 8 \\ \hline 14 - 4 \end{array} \text{ by Prob. VI.}$$

MORE EXAMPLES.

1. From $\frac{8}{12}$ subtract $\frac{5}{12}$. Rem. $\frac{3}{12} = \frac{1}{4}$.
2. From $\frac{8}{9}$ subtract $\frac{2}{3}$. Rem. $\frac{2}{9}$.
3. From $7\frac{5}{8}$ subtract $5\frac{3}{4}$. Rem. $2\frac{1}{4}$.
4. From 1 subtract $\frac{4}{7}$. Rem. $\frac{3}{7}$.
5. From $7\frac{3}{4}$ subtract $5\frac{5}{8}$. Rem. $1\frac{1}{2}$.
6. From $12\frac{7}{8}$ subtract $\frac{5}{8}$. Rem. $12\frac{2}{8}$.
7. From $13\frac{3}{8}$ subtract $\frac{5}{8}$. Rem. $12\frac{5}{8}$.
8. From 50 subtract $9\frac{7}{12}$. Rem. $40\frac{5}{12}$.
9. From 7 subtract $\frac{2}{3}$. Rem. $6\frac{1}{3}$.
10. From $15\frac{3}{4}$ subtract 7. Rem. $8\frac{3}{4}$.
11. From $\frac{8}{9}$ subtract $\frac{2}{3}$ of $\frac{5}{6}$. Rem. $\frac{1}{9}$.

12. From $\frac{2}{3}$ of $\frac{5}{8}$ subtract $\frac{1}{3}$ of $\frac{1}{2}$. Rem. $\frac{4}{21}$.

13. From $\frac{1}{3}$ l. subtract $\frac{7}{8}$ d. Rem. $\frac{1892}{5760}$ l. = 6s. $7\frac{1}{8}$ d.

The reason of the rules in subtraction is the same as in addition. For like things only can be subtracted from one another; and therefore in subtraction the fractions must have all the same denominator, and be of the same denomination.

Multiplication of Vulgar Fractions.

In multiplication of fractions there is no occasion to reduce the given fractions to a common denominator, as in addition and subtraction: only if a mixt number be given, reduce it to an improper fraction; if an integer be given, reduce it to an improper fraction, by putting an unit for its denominator; if a compound fraction be given, you may either reduce it to a simple one; or, instead of the particle *of*, insert the sign of multiplication: then work by the following

R U L E.

Multiply the numerators for the numerator of the product, and multiply the denominators for its denominator.

E X A M P L E S.

$$1. \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}.$$

$$2. \frac{3}{4} \times 5\frac{2}{3} = \frac{3}{4} \times \frac{17}{3} = \frac{51}{12} = 4\frac{3}{12} = 4\frac{1}{4}.$$

$$3. \frac{5}{7} \times 8 = \frac{5}{7} \times \frac{8}{1} = \frac{40}{7} = 5\frac{5}{7}.$$

$$4. \frac{2}{3} \text{ of } \frac{3}{4} \times \frac{5}{8} = \frac{6}{12} \times \frac{5}{8} = \frac{30}{96} = \frac{5}{16}.$$

$$\text{Or, } \frac{2}{3} \times \frac{3}{4} \times \frac{5}{8} = \frac{30}{96} = \frac{5}{16}.$$

N O T E S.

1. If any number be multiplied by a proper fraction, the product will be less than the multiplicand; for multiplication is the taking of the multiplicand as often as the multiplier contains unity; and consequently, if the multiplier be greater than unity, the product will be greater

greater than the multiplicand; if the multiplier be unity, the product will be equal to the multiplicand; and if the multiplier be less than unity, the product will, in the same proportion, be less than the multiplicand. Thus, supposing the multiplier to be $\frac{1}{2}$ or $\frac{1}{3}$, the product, in this case, will be equal to one half or to one third of the multiplicand.

2. Mixt numbers may be multiplied without reducing them to improper fractions, by working as in the margin; where I first multiply the integral parts, *viz.* 54 by 24; then I multiply the integral parts cross-ways into their altern fractions, *viz.* 54 by $\frac{1}{4}$, and the product 27 I set down; in like manner I multiply 24 by $\frac{1}{4}$, and the product 6 I likewise set down; then I add; and to the sum I annex $\frac{1}{8}$, the product of the two fractions.

$$\begin{array}{r} 54\frac{1}{4} \\ 24\frac{1}{4} \\ \hline 216 \\ 108 \\ 27 \\ 6 \\ \hline 1329\frac{1}{8} \end{array}$$

3. In multiplying a fraction by an integer, you have only to multiply the numerator by the integer, the putting 1 for the denominator being only matter of form. And to multiply a fraction by its denominator is to take away the denominator, the product being an integer, the same with, or equal to the numerator. Thus, $\frac{7}{8} \times 8 = 7$. For $\frac{7}{8} \times \frac{8}{1} = \frac{56}{8} = 7$.

4. If the numerators and denominators of two equal fractions be multiplied cross-ways, the products will be equal. Thus, if $\frac{3}{9} = \frac{4}{12}$, then will $3 \times 12 = 9 \times 4$; for multiplying both by 9, we have $3 = \frac{9 \times 4}{12}$; and

multiplying these by 12, we have $3 \times 12 = 9 \times 4$. Hence, if four numbers be proportional, the product of the extremes will be equal to the product of the means: for if $3 : 9 :: 4 : 12$, then $\frac{3}{9} = \frac{4}{12}$; and it has been proved that $3 \times 12 = 9 \times 4$. Therefore if, of four proportional numbers, any three be given, the fourth may easily be found, *viz.* when one of the extremes is sought, divide the product of the means by the given extreme; and when one of the means is sought, divide the product of the extremes by the given mean.

5. In multiplying fractions, equal factors above and below

below may be dashed or dropt. Thus $\frac{1}{2}$ of $\frac{2}{3} \times \frac{3}{4}$ of $\frac{4}{5} = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$; and dropping the factors 2, 3, 4, both above and below, the product is $\frac{1}{5}$. In like manner, to facilitate an operation, a factor above and another below may be divided by the same number: Thus,

$$\frac{6}{7} \times \frac{5}{12} = \frac{1}{7} \times \frac{5}{2} = \frac{5}{7 \times 2} = \frac{5}{14}. \text{ Or we may exchange one numerator for another; Thus, } \frac{6}{7} \times \frac{5}{12} = \frac{6}{12} \times \frac{5}{7} = \frac{1}{2} \times \frac{5}{7} = \frac{5}{14}.$$

6. To take any part of a given number, is to multiply the said number by the fraction. Thus, $\frac{5}{8}$ of 320 is found thus, $\frac{5}{8} \times \frac{320}{1} = \frac{5}{1} \times \frac{320}{8} = \frac{5}{1} \times \frac{40}{1} = \frac{200}{1} = 200$. In like manner, $\frac{2}{3}$ of $45 \frac{3}{8}$, is $\frac{2}{3} \times 45 \frac{3}{8} = \frac{2}{3} \times \frac{363}{8} = \frac{2}{1} \times \frac{603}{4} = \frac{121}{1} = 30 \frac{1}{4}$. Hence, to reduce a compound fraction to a simple one, is to multiply the parts of it into one another.

7. If a multiplicand of two or more denominations be given to be multiplied by a fraction, reduce the higher part or parts of the multiplicand to the lowest species, and then multiply. Thus, to multiply 8l. 10s. $\frac{3}{4}$ by $\frac{2}{3}$, I say, 8l. = 8×20 s. = 160 s. and $160 + 10 \frac{3}{4} = 170 \frac{3}{4}$ s. = $\frac{683}{4}$, and $\frac{2}{3} \times \frac{683}{4} = \frac{1366}{12} = 113 \frac{10}{12}$ s. = L. 5 : 13 : 10. Or, without reducing, you may multiply the given multiplicand by the numerator of the fraction, and divide the product by the denominator.

M O R E E X A M P L E S.

- | | |
|---|-------------------------------------|
| 1. Multiply $\frac{5}{7}$ by $\frac{9}{11}$. | Prod. $\frac{45}{77}$. |
| 2. Multiply $7 \frac{8}{9}$ by $\frac{4}{5}$. | Prod. $6 \frac{14}{5}$. |
| 3. Multiply $8 \frac{2}{3}$ by $9 \frac{3}{4}$. | Prod. $84 \frac{1}{2}$. |
| 4. Multiply $6 \frac{4}{5}$ by 8. | Prod. $54 \frac{2}{5}$. |
| 5. Multiply $9 \frac{1}{2}$ by $\frac{1}{2}$ of $\frac{3}{4}$. | Prod. $48 \frac{9}{16}$. |
| 6. Multiply 12 by $\frac{3}{4}$ of $\frac{4}{5}$. | Prod. $7 \frac{1}{5}$. |
| 7. Multiply $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$. | Prod. $\frac{1}{6}$. |
| 8. Multiply $\frac{13}{18}$ by 18. | Prod. 13. |
| 9. Multiply L. 3 : 12 : $6 \frac{1}{2}$ by $\frac{3}{4}$. | Prod. L. 2 : 14 : $4 \frac{7}{8}$. |

The

The reason of the rule may be shewn thus: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$: for $\frac{4}{5} = \frac{12}{15}$, and $\frac{1}{3}$ of $\frac{12}{15}$ is $\frac{4}{15}$; and consequently $\frac{2}{3}$ of $\frac{12}{15}$ is $\frac{8}{15}$.

The truth of the rule may also be proved thus: Assume two fractions equal to two integers, such as, $\frac{8}{4}$ and $\frac{6}{2}$, equal to 2 and 3, and the product of the fractions will be equal to the product of the integers; for $\frac{8}{4} \times \frac{6}{2} = \frac{48}{8} = 6$, and $2 \times 3 = 6$.

Division of Vulgar Fractions.

In division of fractions, if a mixt number be given, reduce it to an improper fraction; if an integer be given, put an unit for its denominator; if a compound fraction be given, reduce it to a simple one, and then work by the following

R U L E.

Multiply cross-ways, viz. the numerator of the divisor into the denominator of the dividend, for the denominator of the quot; and the denominator of the divisor into the numerator of the dividend, for the numerator of the quot.

E X A M P L E S.

1. $\frac{2}{3}) \frac{4}{5} (\frac{12}{10} = 1 \frac{2}{10} = 1 \frac{1}{5}$.
2. $\frac{3}{4}) 4 \frac{1}{4} (= \frac{3}{4}) \frac{17}{4} (\frac{68}{12} = 5 \frac{8}{12} = 5 \frac{2}{3}$.
3. $4 \frac{2}{3}) \frac{7}{8} (= \frac{14}{3}) \frac{7}{8} (\frac{21}{12} = \frac{3}{8}$.
4. $3 \frac{1}{4}) 6 \frac{5}{8} (= \frac{13}{4}) \frac{53}{8} (\frac{212}{104} = 2 \frac{4}{104} = 2 \frac{1}{26}$.
5. $\frac{4}{5}) 8 (= \frac{4}{5}) \frac{8}{1} (\frac{40}{4} = 10$.
6. $7) \frac{3}{4} (= \frac{7}{1}) \frac{3}{4} (\frac{3}{28}$.
7. $2 \frac{1}{2}) 15 (= \frac{5}{2}) \frac{15}{1} (\frac{30}{5} = 6$.
8. $5) 10 \frac{1}{2} (= \frac{5}{1}) \frac{21}{2} (\frac{21}{10} = 2 \frac{1}{10}$.
9. $\frac{7}{8}) \frac{3}{4} \text{ of } \frac{5}{6} (= \frac{7}{8}) \frac{15}{24} (\frac{120}{168} = \frac{5}{7}$.
10. $\frac{1}{2} \text{ of } \frac{3}{4}) 4 \frac{1}{2} (= \frac{3}{8}) \frac{9}{2} (\frac{72}{6} = 12$.
11. $\frac{2}{3} \text{ of } \frac{4}{5}) 8 (= \frac{8}{15}) \frac{8}{1} (\frac{120}{15} = 15$.
12. $\frac{3}{4} \text{ of } \frac{2}{3} \text{ of } \frac{1}{2}) 4 \text{ of } \frac{5}{6} (= \frac{6}{24}) \frac{20}{3} (\frac{480}{180} = 2 \frac{2}{3}$.

NOTES.

N O T E S.

1. Instead of working division of fractions as taught above, you may invert the divisor, and then multiply it into the dividend. Thus, in Example 1. instead of $\frac{3}{2})\frac{4}{10}(\frac{12}{10}$, you may say, $\frac{3}{2} \times \frac{4}{2} = \frac{12}{2} = 1\frac{2}{2} = 1\frac{1}{1}$.

2. If any number be divided by a proper fraction, the quot will be greater than the dividend: for in division the quot shews how often the divisor is contained in the dividend; and consequently if the divisor be greater than unity, the quot will be less than the dividend; if the divisor be unity, the quot will be equal to the dividend; and if the divisor be less than unity, the quot will, in the same proportion, be greater than the dividend. Thus, supposing the divisor to be $\frac{1}{2}$, or $\frac{1}{3}$, the quot in this case will be double or triple of the dividend.

3. To divide a fraction by an integer, is only to multiply the integer into the denominator of the fraction, the numerator being continued. Thus, $7)\frac{3}{4}(\frac{3}{28}$. See Ex. 6.

4. A mixt number may sometimes be divided by an integer, with more ease, in the following manner. Divide the integral part of the mixt number by the given integer; and if there be no remainder, divide likewise the fraction of the mixt number by the given integer, and annex the quot to the integral quot formerly found. Thus, in working Ex. 8. by this method, I say, $5)10(2$, and $5)\frac{1}{2}(\frac{1}{10}$; and so the complete quot is $2\frac{1}{10}$, as before. But if, in dividing the integral part, there happen to be a remainder, prefix this remainder to the fraction for a new mixt number; which reduce to an improper fraction: then divide the improper fraction by the given integer; and annex the quot to the integral quot formerly found. Thus, if it be required to divide $15\frac{3}{4}$ by 8, I say, $8)15(1$, and 7 remains; which 7, prefixed to the fraction, gives $7\frac{3}{4}$ for a new mixt number; and this, reduced to an improper fraction, is $\frac{31}{4}$, and $8)\frac{31}{4}(\frac{31}{4}$: so the complete quot is $1\frac{31}{4}$.

5. If the factors of the numerator and denominator of the quots, instead of being actually multiplied, be only connected with the sign of multiplication, it will be
easy

easy to drop such factors, above and below, as happen to be the same, thus: $\frac{3}{4} \div \frac{4}{5}$ of $\frac{3}{8} \left(\frac{4 \times 4 \times 3}{3 \times 5 \times 8} = \frac{4 \times 4}{5 \times 8} = \frac{16}{40} = \frac{2}{5} \right)$. Or a factor above and below may be divided by the same number, thus: $\frac{5}{8} \div \frac{7}{12} \left(\frac{6 \times 7}{5 \times 12} = \frac{7}{5 \times 2} = \frac{7}{10} \right)$. Or the factors of the numerator of the quot may be exchanged, thus: $\frac{2}{3} \div \frac{5}{9} \left(\frac{3 \times 5}{2 \times 9} = \frac{5 \times 3}{2 \times 9} = \frac{5}{2 \times 3} = \frac{5}{6} \right)$.

6. To divide an integer by a fraction, is to divide the product of the denominator and integer by the numerator, thus: $\frac{4}{5} \div 8 \left(= \frac{5 \times 8}{4} = 5 \times 2 = 10 \right)$. See Ex. 5.

7. If the divisor and dividend have the same denominator, you have only to divide the numerator of the dividend by the numerator of the divisor, thus: $\frac{5}{8} \div \frac{3}{8} (= \frac{5}{3})$; for $\frac{5}{8} \div \frac{3}{8} \left(\frac{8 \times 3}{5 \times 8} = \frac{3}{5} \right)$.

8. If a dividend of two or more denominations be given to be divided by a fraction, reduce the higher part or parts of the dividend to the lowest species, and then divide. Thus, to divide 6l. 9s. $\frac{3}{4}$ by $\frac{2}{3}$, I say, 6l. = 6×20 s. = 120; and $120 + 9 \frac{3}{4} = 129 \frac{3}{4}$ s. = $\frac{512}{4}$; and $\frac{2}{3} \div \frac{512}{4} \left(\frac{1536}{3} = 512 \right) = 194 \frac{5}{8}$ s. = 9l. 14s. $7 \frac{1}{2}$ d.

Or, Divide the given multiplicand by the numerator of the fraction, and multiply the quot by the denominator.

E X A M P L E.

Divide L. 276 : 16 : 8 among four men, A, B, C, D, so that A, B, C, may have equal shares, and D only two thirds of one of their shares.

$$1 + 1 + 1 + \frac{2}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3} = \frac{11}{3}$$

	L.	s.	d.	L.	s.	d.	L.	s.	d.	
11) 276	16	8	(25	3	4	$\times 3 = 75$	10			A.
						$\times 3 = 75$	10			B.
						$\times 3 = 75$	10			C.
						$\times 2 = 50$	6	8		D.

Proof 276 16 8

H h

M O R E

MORE EXAMPLES.

1. By $\frac{3}{8}$ divide $\frac{5}{8}$. Quot $\frac{25}{24} = 1 \frac{1}{24}$.
2. By $3 \frac{3}{4}$ divide $2 \frac{2}{5}$. Quot $\frac{16}{25}$.
3. By 9 divide $19 \frac{1}{2}$. Quot $2 \frac{1}{6}$.
4. By $5 \frac{2}{3}$ divide 18. Quot $3 \frac{3}{17}$.
5. By $7 \frac{1}{2}$ divide $\frac{4}{5}$ of $\frac{5}{8}$. Quot $\frac{4}{45}$.
6. By $\frac{3}{4}$ divide 12 l. 6 s. Quot $\frac{11840}{3}$ d. = 16 l. 8 s.
8 d. 10 $\frac{2}{3}$ d.

The reason of the rule will appear by considering, that the method here used is nothing else but the reducing the divisor and dividend to a common denominator, and then dividing the one numerator by the other. Thus, $\frac{3}{4} \div \frac{2}{3}$ ($\frac{8}{9}$, for reducing the divisor and dividend to a common denominator, we have $\frac{2}{12} \div \frac{8}{12}$ ($= \frac{8}{9}$).

The truth of the rule may also be proved by assuming two fractions equal to two integers, such as, $\frac{6}{3}$ and $\frac{16}{4}$, equal to 2 and 4, and the quot of the fractions will be equal to the quot of the integers. Thus, $\frac{6}{3} \div \frac{16}{4}$ ($\frac{48}{24} = 2$, and $2 \div 4$ (2).

Practical Questions.

Quest. 1. The lesser of two numbers is $36 \frac{8}{9}$; their difference is $11 \frac{25}{6}$: What is the greatest? *Ans.* $48 \frac{7}{12}$.

Quest. 2. What number added to $4 \frac{7}{8}$, will give $12 \frac{4}{5}$? *Ans.* $7 \frac{37}{40}$.

Quest. 3. A has $\frac{7}{10}$ of a ship, and B has $\frac{2}{5}$ of the same vessel: Which of them has the greatest share? and what the difference? *Ans.* A has the greatest share; and the difference of their shares is $\frac{1}{10}$.

Quest. 4. Three purses contain $130 \frac{2}{5}$ l.; in one purse is $55 \frac{5}{8}$ l. in another $42 \frac{4}{5}$ l.: How much is in the third purse? *Ans.* $32 \frac{1}{3}$ l.

Quest. 5. What is $\frac{5}{8}$ parts of $130 \frac{2}{3}$? *Ans.* $81 \frac{2}{3}$.

Quest. 6. What number multiplied by $\frac{3}{5}$ will produce $25 \frac{2}{3}$? *Ans.* $42 \frac{1}{3}$.

The

The Simple Rule of Three in Vulgar Fractions.

The question is stated as formerly taught in the rule of three. The extremes must be of one denomination. Reduce mixt numbers and integers to improper fractions, compound fractions to simple ones, and then work by the following rule, *viz.*

Multiply the second and third terms, and divide the product by the first term: that is, multiply the numerator of the first term into the denominators of the second and third, for the denominator of the answer; and multiply the denominator of the first term into the numerators of the second and third, for the numerator of the answer.

I. *Direct Questions.*

Quest. 1. If $\frac{3}{4}$ yard cost $\frac{5}{8}$ l. what will $\frac{9}{10}$ yard cost?

$$\begin{array}{ccc} \text{rd.} & \text{L.} & \text{rd.} \\ \text{If } \frac{3}{4} : \frac{5}{8} :: \frac{9}{10}. \end{array}$$

$$\text{Ans. } \frac{4 \times 5 \times 9}{3 \times 8 \times 10} = \frac{4 \times 9}{3 \times 2 \times 10} = \frac{9}{3 \times 2 \times 2} = \frac{3}{2 \times 2} =$$

$$\frac{3}{4} \text{ l.} = \frac{3 \times 20}{4} \text{ s.} = \frac{60}{4} = 15 \text{ s.}$$

Quest. 2. If $\frac{5}{6}$ yard cost $\frac{2}{3}$ l. what will $\frac{11}{12}$ yard cost?

$$\begin{array}{ccc} \text{rd.} & \text{L.} & \text{rd.} \\ \text{If } \frac{5}{6} : \frac{2}{3} :: \frac{11}{12}. \end{array}$$

$$\text{Ans. } \frac{6 \times 2 \times 11}{5 \times 3 \times 12} = \frac{2 \times 11}{5 \times 3 \times 2} = \frac{11}{5 \times 3} = \frac{11}{15} \text{ l.} =$$

$$\frac{11 \times 20}{15} \text{ s.} = \frac{220}{15} = 14 \text{ s. } 8 \text{ d.}$$

Quest. 3. If 3 yards cost $2\frac{4}{5}$ l. what will $14\frac{3}{4}$ yards cost?

H h 2

Yds.

$$\begin{array}{rcl} \text{Yds.} & \text{L.} & \text{Yds.} \\ \text{If } 3 & : 2\frac{4}{5} & :: 14\frac{3}{4} \\ & \frac{3}{1} : \frac{14}{5} & :: \frac{52}{4} \end{array}$$

$$\text{Ans. } \frac{1 \times 14 \times 59}{3 \times 5 \times 4} = \frac{7 \times 59}{3 \times 5 \times 2} = \frac{413}{30} \text{ l.} = 13 - 15 - 4.$$

MORE EXAMPLES.

Quest. 4. If $2\frac{3}{4}$ lb. tobacco cost $3\frac{1}{2}$ s. what will $242\frac{1}{4}$ lb cost? *Ans.* L. 15 : 8 : $3\frac{3}{4}$.

Quest. 5. If $\frac{3}{4}$ of $\frac{3}{8}$ C. sugar cost $\frac{9}{10}$ l. what will 82 C. cost? *Ans.* 262 l. 8 s.

Quest. 6. A mercer bought $3\frac{1}{2}$ pieces of silk, each piece containing $24\frac{2}{3}$ ells, at 6 s. $\frac{3}{4}$ d. *per ell*: Required the value of the $3\frac{1}{2}$ pieces at that rate? *Ans.* 26 l. 3 s. 4 $\frac{3}{4}$ d.

Quest. 7. If $\frac{2}{5}$ oz. silver cost 2 s. what will be the price of $11\frac{2}{3}$ lb. at that rate? *Ans.* 35 l.

Quest. 8. If $1\frac{5}{7}$ lb. of gold is worth 61 $\frac{5}{7}$ l. Sterling, what is a grain worth at that rate? *Ans.* 1 $\frac{1}{2}$ d.

Quest. 9. If $\frac{3}{4}$ yard of silk cost $\frac{3}{4}$ of $\frac{5}{8}$ l. what is the price of $15\frac{2}{3}$ ells Flemish? *Ans.* L. 9 : 15 : 10.

Quest. 10. If $\frac{2}{3}$ of $\frac{3}{4}$ lb. of cloves cost 6 s. 2 $\frac{2}{7}$ d. what cost the C. at that rate? *Ans.* L. 69 : 6 : 8.

II. Inverse Questions.

Quest. 1. If $\frac{3}{4}$ yard of cloth that is 2 yards wide, will make a garment, how much of any other cloth that is $\frac{3}{5}$ yard wide will make the same garment?

$$\begin{array}{rcl} \text{Bread.} & \text{len.} & \text{Bread.} \\ \frac{3}{5} & : \frac{3}{4} & :: \frac{2}{1} \end{array}$$

$$\text{Ans. } \frac{5 \times 1 \times 2}{3 \times 4 \times 1} = \frac{5 \times 2}{4} = \frac{5}{2} = 2\frac{1}{2} \text{ yards.}$$

Quest. 2. If I lend my friend 48 l. for $\frac{3}{4}$ of a year, how much ought he to lend me for $\frac{5}{12}$ of a year?

Yea.

$$\begin{array}{ccc} \text{Yea.} & \text{L.} & \text{Yea.} \\ \frac{5}{12} & : \frac{48}{1} & :: \frac{3}{4}. \end{array}$$

$$\text{Ans. } \frac{12 \times 48 \times 3}{5 \times 1 \times 4} = \frac{12 \times 12 \times 3}{5} = \frac{432}{5} = 86\frac{1}{5} \text{ s.}$$

MORE EXAMPLES.

Quest. 3. If $\frac{2}{3}$ yard of cloth that is $2\frac{1}{3}$ yards wide, will make a coat, what is the breadth of the cloth whereof $1\frac{3}{4}$ yard will make the same coat? *Ans.* $\frac{56}{83}$ yard, or 3 qrs $2\frac{2}{3}$ nails.

Quest. 4. How many inches in length, of a plank that is 9 inches broad, will make a foot square? *Ans.* 16 inches in length.

Quest. 5. If the penny-loaf weigh $10\frac{2}{3}$ ounces when the bushel of wheat costs $4\frac{3}{4}$ s. what ought the penny-loaf to weigh when the bushel of wheat costs $8\frac{9}{16}$ s.?

Ans. $5\frac{185}{267}$ ounces.

Quest. 6. If 12 men do a piece of work in $10\frac{2}{3}$ days, in how many days will 6 men do the same? *Ans.* In $21\frac{1}{3}$ days.

The Compound Rule of Three in Vulgar Fractions.

Quest. 1. If $\frac{3}{4}$ acre of grass be cut down by 2 men in $\frac{2}{3}$ day, how many acres shall be cut down by 6 men in $3\frac{1}{3}$ days?

$$\begin{array}{ccc} \text{Men.} & \text{acr.} & \text{Men.} \\ \frac{2}{1} & : \frac{3}{4} & :: \frac{6}{1}. \end{array}$$

$$\text{day } \frac{2}{3} \qquad 3\frac{1}{3} = \frac{10}{3} \text{ days.}$$

$$\frac{4}{3} : \frac{3}{4} :: \frac{60}{3}.$$

$$\text{Ans. } \frac{3 \times 3 \times 60}{4 \times 4 \times 3} = \frac{3 \times 60}{4 \times 4} = \frac{3 \times 15}{4} = \frac{45}{4} = 11\frac{1}{4} \text{ acr.}$$

Or thus :

$$\begin{array}{l} \text{Ans. } \frac{3 \times 1 \times 3 \times 6 \times 10}{2 \times 2 \times 4 \times 1 \times 3} = \frac{3 \times 6 \times 10}{2 \times 2 \times 4} = \frac{3 \times 3 \times 5}{2 \times 2} = \\ \frac{45}{4} = 11\frac{1}{4} \text{ acres.} \end{array}$$

Quest.

Quest. 2. If 4 men cast a ditch $8\frac{1}{2}$ feet long, $3\frac{1}{3}$ deep, and $2\frac{1}{4}$ broad, in $9\frac{1}{3}$ days, in how many days will 8 men cast a ditch $25\frac{1}{2}$ feet long, $6\frac{2}{3}$ deep, and $4\frac{1}{2}$ broad?

		<i>Men.</i>		<i>days.</i>		<i>Men.</i>	
<i>b.</i>	<i>d.</i>	<i>* 8</i>	<i>:</i>	$9\frac{1}{3}$	<i>::</i>	<i>4*</i>	<i>d.</i>
$2\frac{1}{4}$	$3\frac{1}{3}$	$1.8\frac{1}{2}$				$25\frac{1}{2}$	$6\frac{2}{3}$
		$\frac{8}{1}$	<i>:</i>	$2\frac{8}{3}$	<i>::</i>	$\frac{4}{1}$	
$\frac{9}{4}$	$\frac{10}{3}$	$\frac{17}{2}$				$\frac{51}{2}$	$\frac{20}{3}$

$$\frac{9 \times 10 \times 17 \times 8}{4 \times 3 \times 2 \times 1} = \frac{2 \times 5 \times 17 \times 2}{1} = \frac{310}{1} \text{ first term.}$$

$$\frac{4 \times 51 \times 20 \times 9}{1 \times 2 \times 3 \times 4} = \frac{2 \times 51 \times 10 \times 3}{1} = \frac{3060}{1} \text{ third term.}$$

$$\frac{310}{1} : \frac{28}{3} :: \frac{3060}{1}$$

$$\frac{1 \times 28 \times 3060}{510 \times 3 \times 1} = \frac{14 \times 1020}{255} = \frac{14280}{255} = 56 \text{ days.}$$

Or thus:

$$\frac{4 \times 2 \times 2 \times 1 \times 28 \times 4 \times 51 \times 20 \times 9}{9 \times 10 \times 17 \times 8 \times 3 \times 1 \times 2 \times 3 \times 2} =$$

$$\frac{4 \times 3 \times 2 \times 28 \times 4 \times 51 \times 20}{10 \times 7 \times 8 \times 3 \times 2 \times 3 \times 2} = \frac{2 \times 28 \times 1 \times 51 \times 2}{17 \times 2 \times 3 \times 2} =$$

$$\frac{28 \times 4 \times 51}{17 \times 3 \times 2} = \frac{14 \times 4 \times 51}{17 \times 3} = \frac{14 \times 3 \times 17}{17} = \frac{14 \times 4}{1} =$$

$$\frac{56}{1} = 56 \text{ days.}$$

C H A P. X.

Rules of Practice.

WHEN the first term of a question in the rule of three happens to be unity, the answer may frequently be found more speedily and easily than by a formal stating or working of the rule of three; and the directions to be observed in such operations are called *Rules of Practice*.

The rules of practice naturally follow the doctrine of vulgar

vulgar fractions, the operation being nothing else but a multiplying the number whose price is required, by such a fraction of a pound, of a shilling, or of a penny, as denotes the rate or price of one.

Thus, if the price of 24 yards, at 6s. 8d. *per* yard, be demanded, the answer is found by multiplying 24 by $\frac{1}{3}$, the fraction of a pound equivalent to 6s. 8d. *viz.* $\frac{24}{1} \times \frac{1}{3} = \frac{24}{3} = 8$ l.

Hence it is obvious, that to multiply a number by a fraction whose numerator is unity, is to divide the said number by the denominator of the fraction. But if the numerator of the fraction be not unity, you must first multiply the given number by the numerator, and then divide the product by the denominator. Thus, if the rate be 13s. 4d. $= \frac{2}{3}$ l. the price of 24 yards is found by saying, $\frac{24}{1} \times \frac{2}{3} = \frac{48}{3} = 16$ l.; or take $\frac{1}{3}$ of the given number twice.

When the fraction denoting the rate happens to be compound, the product or answer is found by dividing the given number by one of the denominators of the compound fraction, the quot by another, and the next quot by the third, &c. Thus, if the rate be 2 farthings $= \frac{1}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ l. the price of 1440 yards is found by saying, $\frac{1440}{2} = 720$, and $\frac{720}{12} = 60$, and $\frac{60}{20} = 3$ l.

When the rate is expressed by two or more simple fractions, connected with the sign $+$, the product or answer is found by dividing the given number successively by the several denominators, and then adding the quot. Thus, if the rate be 3s. $= \frac{1}{10} + \frac{1}{20}$ l. the price of 80 yards is found by saying, $\frac{80}{10} = 8$, and $\frac{80}{20} = 4$, and $8 + 4 = 12$ l.

The fractions equivalent to any number of farthings under 4, to any number of pence under 12, and to any number of shillings under 20, are exhibited in the following tables.

T A B L E

T A B L E I.

Farthings.	of a penny.	of a shilling.	of a pound.
1	$\frac{1}{4}$	$\frac{1}{4}$ of $\frac{1}{12}$	$\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$
2	$\frac{1}{2}$	$\frac{1}{2}$ of $\frac{1}{12}$	$\frac{1}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$
3	$\frac{3}{4}$	$\frac{3}{4}$ of $\frac{1}{12}$	$\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$

TABLE II.

Pen.	of a shill.
1	$\frac{1}{12}$
1 $\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{6}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
5	$\frac{1}{4} + \frac{1}{6}$
6	$\frac{1}{2}$
7	$\frac{1}{3} + \frac{1}{4}$
8	$\frac{1}{3} + \frac{1}{3}$
9	$\frac{1}{2} + \frac{1}{4}$
10	$\frac{1}{2} + \frac{1}{3}$
11	$\frac{1}{3} + \frac{1}{3} + \frac{1}{4}$

TABLE III.

s. d.	of a pound.
1	$\frac{1}{20}$
1 8	$\frac{1}{12}$
2	$\frac{1}{10}$
2 6	$\frac{1}{8}$
3	$\frac{1}{10} + \frac{1}{20}$
3 4	$\frac{1}{6}$
4	$\frac{2}{10}$, or $\frac{1}{5}$
5	$\frac{2}{10} + \frac{1}{20}$, or $\frac{1}{4}$
6	$\frac{3}{10}$
6 8	$\frac{1}{3}$
7	$\frac{3}{10} + \frac{1}{20}$
8	$\frac{4}{10}$, or $\frac{2}{5}$
9	$\frac{4}{10} + \frac{1}{20}$
10	$\frac{5}{10}$, or $\frac{1}{2}$
11	$\frac{5}{10} + \frac{1}{20}$
12	$\frac{6}{10}$, or $\frac{3}{5}$
13	$\frac{6}{10} + \frac{1}{20}$
13 4	$\frac{2}{3}$
14	$\frac{7}{10}$
15	$\frac{7}{10} + \frac{1}{20}$, or $\frac{3}{4}$
16	$\frac{8}{10}$, or $\frac{4}{5}$
17	$\frac{8}{10} + \frac{1}{20}$
18	$\frac{9}{10}$
19	$\frac{9}{10} + \frac{1}{20}$

The fractions in Table II. become compound fractions of a pound, by annexing (of $\frac{1}{20}$) to each of them. Thus, 1 d. is $\frac{1}{12}$ of $\frac{1}{20}$ l.; and 5 d. is $\frac{1}{4}$ of $\frac{1}{20}$ + $\frac{1}{6}$ of $\frac{1}{20}$ l. &c.

The variety that occurs in the rules of practice arises chiefly from the different rates, or prices, of one thing, as a yard, a pound, an ounce, &c. and may be reduced to the eight cases following, viz.

The rate may be, 1. Farthings under four. 2. Pence under twelve. 3. Pence and farthings. 4. Shillings under twenty. 5. Shillings, pence, and farthings. 6. Pounds. 7. Pounds, shillings, pence, and farthings. 8. The given number may consist of integers and parts.

C A S E

C A S E I.

When the rate is farthings, under four.

R U L E.

Divide the given number by the denominator of the fraction denoting the rate, as contained in Table I. viz. if the rate be 1 or 2 farthings, divide by 4 or 2, the quot will be pence; and the remainder, in dividing by 4, will be farthings, and in dividing by 2, it will be 1 halfpenny: then divide the pence by 12, the quot will be shillings, and the remainder pence: lastly, divide the shillings by 20, the quot will be pounds, and the remainder shillings. But if the rate is 3 farthings, first multiply the given number by the numerator 3, and then divide as above directed.

E X A M P L E S.

Ex. 1.

$$\begin{array}{r} \frac{1}{4} \overline{) 4859, \text{ at } 1 \text{ f.}} \\ \frac{1}{12} \overline{) 1214 - 3 \text{ f.}} \\ \frac{1}{20} \overline{) 101 - 2 \text{ d.}} \\ \hline \text{L. } 5 \quad 1 \quad 2 \frac{3}{4}. \end{array}$$

Ex. 2.

$$\begin{array}{r} \frac{1}{2} \overline{) 8347, \text{ at } 2 \text{ f.}} \\ \frac{1}{12} \overline{) 4173 - \frac{1}{2} \text{ d.}} \\ \frac{1}{20} \overline{) 347 - 9 \text{ d.}} \\ \hline \text{L. } 17 \quad 7 \quad 9 \frac{1}{2} \end{array}$$

Ex. 3.

$$\begin{array}{r} 1753, \text{ at } 3 \text{ f.} \\ 3 \\ \hline \frac{3}{4} \overline{) 5259} \\ \frac{1}{12} \overline{) 1314 - 3 \text{ f.}} \\ \frac{1}{20} \overline{) 109 - 6 \text{ d.}} \\ \hline \text{L. } 5 \quad 9 \quad 6 \frac{3}{4}. \end{array}$$

Or, because $3 \text{ f.} = \frac{1}{2} \text{ d.} + \frac{1}{4} \text{ d.}$ or $= \frac{1}{2} \text{ d.} + \frac{1}{2} \text{ of } \frac{1}{2} \text{ d.}$ you may work Ex. 3. either of the two ways following.

$$\begin{array}{r} 1753, \text{ at } 3 \text{ f.} \\ \frac{1}{2} \overline{) 876 - 2 \text{ f.}} \\ \frac{1}{4} \overline{) 438 - 1 \text{ f.}} \\ \frac{1}{12} \overline{) 1314 - 3 \text{ f.}} \\ \frac{1}{20} \overline{) 109 - 6 \text{ d.}} \\ \hline \text{L. } 5 \quad 9 \quad 6 \frac{3}{4}. \\ \text{I } i \end{array}$$

$$\begin{array}{r} 1753, \text{ at } 3 \text{ f.} \\ \frac{1}{2} \overline{) 876 - 2 \text{ f.}} \\ \frac{1}{4} \overline{) 438 - 1 \text{ f.}} \\ \frac{1}{12} \overline{) 1314 - 3} \\ \frac{1}{20} \overline{) 109 - 6} \\ \hline \text{L. } 5 \quad 9 \quad 6 \frac{3}{4}. \\ \text{More} \end{array}$$

More ways of working the above examples might be assigned, but the most easy and simple methods are the best.

C A S E II.

When the rate is pence, under twelve.

R U L E.

Divide the given number by the denominator of the fraction denoting the rate, as contained in Table II. and you have the answer in shillings; which reduce into pounds, by dividing by 20.

E X A M P L E S.

$$\begin{array}{r}
 \text{Ex. 1.} \\
 \frac{\frac{1}{12}}{818, \text{ at } 1 \text{ d.}} \\
 \hline
 \frac{\frac{1}{20}}{68-2 \text{ d.}} \\
 \hline
 \text{L. } 3 \quad 8 \quad 2
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 2.} \\
 \frac{\frac{1}{6}}{5316, \text{ at } 2 \text{ d.}} \\
 \hline
 \frac{\frac{1}{20}}{88|6} \\
 \hline
 \text{L. } 44 \quad 6
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 3.} \\
 \frac{\frac{1}{4}}{879, \text{ at } 3 \text{ d.}} \\
 \hline
 \frac{\frac{1}{20}}{21|9-9} \\
 \hline
 \text{L. } 10 \quad 19 \quad 9
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 4.} \\
 \frac{\frac{1}{3}}{3097, \text{ at } 4 \text{ d.}} \\
 \hline
 \frac{\frac{1}{20}}{103|2-4} \\
 \hline
 \text{L. } 51 \quad 12 \quad 4
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 5.} \\
 439, \text{ at } 5 \text{ d.} \\
 \hline
 \begin{array}{l}
 3 \text{ d. } \frac{\frac{1}{4}}{109-9} \\
 2 \text{ d. } \frac{\frac{1}{6}}{73-2} \\
 \hline
 \frac{\frac{1}{20}}{18|2-11} \\
 \hline
 \text{L. } 9 \quad 2 \quad 11
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 6.} \\
 \frac{\frac{1}{2}}{78642, \text{ at } 6 \text{ d.}} \\
 \hline
 \frac{\frac{1}{20}}{3932|1} \\
 \hline
 \text{L. } 1966 \quad 1
 \end{array}$$

Ex.

Ex. 7.
587, at 7 d.
 4 d. $\frac{1}{3}$ 195—8
 3 d. $\frac{1}{4}$ 146—9

 $\frac{1}{20}$ 34|2—5

 L. 17 2 5

Ex. 8.
836, at 8 d.
 4 d. $\frac{1}{3}$ 278—8
 4 d. $\frac{1}{3}$ 278—8

 $\frac{1}{20}$ 55|7—4

 L. 27 17 4

Ex. 9.
417, at 9 d.
 6 d. $\frac{1}{2}$ 208—6
 3 d. $\frac{1}{4}$ 104—3

 $\frac{1}{20}$ 31|2—9

 L. 15 12 9

Ex. 10.
386, at 10 d.
 6 d. $\frac{1}{2}$ 193
 4 d. $\frac{1}{3}$ 128—8

 $\frac{1}{20}$ 32|1—8

 L. 16 1 8

Ex. 11.
534, at 11 d.
 4 d. $\frac{1}{3}$ 178
 4 d. $\frac{1}{3}$ 178
 3 d. $\frac{1}{4}$ 133—6

 $\frac{1}{20}$ 48|9—6

 L. 24 9 6

Note 1. The remainders at the first division in all the above examples is the same with the rate. Thus, in Ex. 1. every remainder is 1 d.; in Ex. 3. every remainder is 3 d.; and in Ex. 5. every remainder, in dividing by 4, is 3 d.; and, in dividing by 6, every remainder is 2 d. &c.

Note 2. In all the examples wherein the rate is no aliquot part of a shilling, the question may be solved as
 I i 2 many

many different ways as you can find different fractions equivalent to the rate. Thus, 5 d. = $\frac{1}{3}$ s. + $\frac{1}{12}$ s. or $= \frac{1}{3}$ s. + $\frac{1}{4}$ of $\frac{1}{3}$ s. Or 5 d. = $\frac{1}{6}$ of half a crown, and 1 half crown = $\frac{1}{8}$ l.; that is, 5 d. = $\frac{1}{6}$ of $\frac{1}{8}$ l.; and accordingly Ex. 5. may also be solved the three ways following.

	439, at 5 d. $\frac{1}{3}$		439. at 5 d. $\frac{1}{6}$		439, at 5 d.
$\frac{1}{3}$	146—4	$\frac{1}{4}$	146—4	$\frac{1}{8}$	73 h. er. 5 d.
$\frac{1}{12}$	36—7		36—7		
					L. 9 2 11
$\frac{1}{20}$	18 2—11	$\frac{1}{20}$	18 2—11		
	L 9 2 11		L. 9 2 11		

In like manner, 9 d. = $\frac{1}{2}$ s. + $\frac{1}{2}$ of $\frac{1}{2}$ s. or 9 d. = $\frac{1}{40}$ l. + $\frac{1}{2}$ of $\frac{1}{40}$ l.; and accordingly Ex. 9. may also be solved other two ways, as under.

	417, at 9 d. $\frac{1}{2}$		41 7, at 9 d. $\frac{1}{40}$
$\frac{1}{2}$	208—6	$\frac{1}{2}$	10—8—6
	104—3		5—4—3
$\frac{1}{20}$	31 2—9		L. 15 12 9
	L. 15 12 9		

Note 3. When the rate is 11 d. you may work as if it were 12 d.; that is, from the given number you may subtract $\frac{1}{12}$ of itself, and the remainder will be the price in shillings. Thus, $\frac{534 \text{ s.}}{12} = 44 \text{ s. 6 d.}$ and $534 \text{ s.} - 44 \text{ s. 6 d.} = 489 \text{ s. 6 d.} = \text{L. } 24 : 9 : 6$. See Ex. 11.

C A S E III.

When the rate is pence and farthings.

R U L E.

R U L E.

The pence must be some aliquot part of a shilling, and at the same time the farthings some aliquot part of the pence; and if they be not so given, divide the pence into two or more such parts, so as the farthings may be some aliquot part of the lowest division of the pence. Then, beginning with the highest division of the pence, divide by the denominators of the fractions denoting the aliquot parts.

E X A M P L E S.

Ex. 1.

1 d.	$\frac{1}{12}$	532, at $1\frac{1}{4}$ d.
$\frac{1}{4}$ d.	$\frac{1}{4}$	44—4
		11—1
	$\frac{1}{20}$	5 5—5
		L. 2 15 5

Ex. 2.

1 d.	$\frac{1}{12}$	1753, at $1\frac{1}{2}$ d.
$\frac{1}{2}$ d.	$\frac{1}{2}$	146—1
		73— $0\frac{1}{2}$
	$\frac{1}{20}$	21 9— $1\frac{1}{2}$
		L. 10 19 $1\frac{1}{2}$

Or rather thus.

$1\frac{1}{2}$ d.	$\frac{1}{8}$	1753, at $1\frac{1}{2}$ d.
	$\frac{1}{20}$	21 9— $1\frac{1}{2}$
		L. 10 19 $1\frac{1}{2}$

Ex. 3.

$1\frac{1}{2}$ d.	$\frac{1}{8}$	1753, at $1\frac{3}{4}$ d.
$\frac{1}{4}$ d.	$\frac{1}{8}$	219— $1\frac{1}{2}$
		36— $6\frac{1}{4}$
	$\frac{1}{20}$	25 5— $7\frac{3}{4}$
		L. 12 15 $7\frac{3}{4}$

Ex. 4.

1 d.	$\frac{1}{12}$	859, at $1\frac{1}{8}$ d.
$\frac{1}{8}$ d.	$\frac{1}{8}$	71—7
		8—11— $1\frac{1}{2}$ f.
	$\frac{1}{20}$	8 0—6— $1\frac{1}{2}$
		L. 4 0 6 $1\frac{1}{2}$ f.

Ex. 5.

2 d.	$\frac{1}{8}$	485, at $2\frac{1}{4}$ d.
$\frac{1}{4}$ d.	$\frac{1}{8}$	80—10
		10— $1\frac{1}{4}$
	$\frac{1}{20}$	9 0— $11\frac{1}{4}$
		L. 4 10 $11\frac{1}{4}$

Ex.

Ex. 6.

$$\begin{array}{r}
 3 \text{ d. } \frac{1}{4} \quad 3471, \text{ at } 3\frac{1}{2} \text{ d.} \\
 \frac{1}{2} \text{ d. } \frac{1}{6} \quad 867-9 \\
 \quad \quad 144-7\frac{1}{2} \\
 \hline
 \frac{1}{20} \quad 101|2-4\frac{1}{2} \\
 \hline
 \text{L. } 50 \quad 12 \quad 4\frac{1}{2}
 \end{array}$$

Ex. 7.

$$\begin{array}{r}
 4 \text{ d. } \frac{1}{3} \quad 475, \text{ at } 4\frac{3}{4} \text{ d.} \\
 \frac{1}{2} \text{ d. } \frac{1}{8} \quad 158-4 \\
 \frac{1}{4} \text{ d. } \frac{1}{2} \quad 19-9\frac{1}{2} \\
 \quad \quad 9-10\frac{3}{4} \\
 \hline
 \frac{1}{20} \quad 18|8-0\frac{1}{4} \\
 \hline
 \text{L. } 9 \quad 8 \quad 0\frac{1}{4}
 \end{array}$$

Ex. 8.

$$\begin{array}{r}
 976, \text{ at } 5\frac{1}{2} \text{ d.} \\
 \hline
 4 \text{ d. } \frac{1}{3} \quad 325-4 \\
 1\frac{1}{2} \text{ d. } \frac{1}{8} \quad 122 \\
 \hline
 \frac{1}{20} \quad 44|7-4 \\
 \hline
 \text{L. } 22 \quad 7 \quad 4
 \end{array}$$

Ex. 9.

$$\begin{array}{r}
 3506, \text{ at } 6\frac{1}{4} \text{ d.} \\
 \hline
 3 \text{ d. } \frac{1}{4} \quad 876-6 \\
 3 \text{ d. } \frac{1}{4} \quad 876-6 \\
 \frac{1}{4} \text{ d. } \frac{1}{12} \quad 73-0\frac{1}{2} \\
 \hline
 \frac{1}{20} \quad 182|6-0\frac{1}{2} \\
 \hline
 \text{L. } 91 \quad 6 \quad 0\frac{1}{2}
 \end{array}$$

Ex. 10.

$$\begin{array}{r}
 520, \text{ at } 7\frac{3}{4} \text{ d.} \\
 \hline
 4 \text{ d. } \frac{1}{3} \\
 3 \text{ d. } \frac{1}{4} \quad 173-4 \\
 \frac{3}{4} \text{ d. } \frac{1}{4} \quad 130 \\
 \quad \quad 22-6 \\
 \hline
 \frac{1}{20} \quad 54|8-10 \\
 \hline
 \text{L. } 16 \quad 15 \quad 10
 \end{array}$$

Ex. 11.

$$\begin{array}{r}
 782, \text{ at } 8\frac{1}{4} \text{ d.} \\
 \hline
 6 \text{ d. } \frac{1}{2} \\
 2 \text{ d. } \frac{1}{3} \quad 391-6 \\
 \frac{1}{4} \text{ d. } \frac{1}{8} \quad 130-6 \\
 \quad \quad 16-3\frac{3}{4} \\
 \hline
 \frac{1}{20} \quad 53|8-3\frac{3}{4} \\
 \hline
 \text{L. } 26 \quad 18 \quad 3\frac{3}{4}
 \end{array}$$

Ex. 12.

$$\begin{array}{r}
 7012, \text{ at } 9\frac{3}{4} \text{ d.} \\
 \hline
 6 \text{ d. } \frac{1}{2} \\
 3 \text{ d. } \frac{1}{2} \quad 3506 \\
 \frac{3}{4} \text{ d. } \frac{1}{4} \quad 1753 \\
 \quad \quad 438-3 \\
 \hline
 \frac{1}{20} \quad 569|7-3 \\
 \hline
 \text{L. } 284 \quad 17 \quad 3
 \end{array}$$

Ex. 13.

$$\begin{array}{r}
 137, \text{ at } 10\frac{1}{2} \text{ d.} \\
 \hline
 6 \text{ d. } \frac{1}{2} \\
 3 \text{ d. } \frac{1}{2} \quad 68-6 \\
 1\frac{1}{2} \text{ d. } \frac{1}{2} \quad 34-3 \\
 \quad \quad 17-1\frac{1}{2} \\
 \hline
 \frac{1}{20} \quad 11|9-10\frac{1}{2} \\
 \hline
 \text{L. } 5 \quad 19 \quad 10\frac{1}{2}
 \end{array}$$

Ex. 14.

6 d.	$\frac{1}{2}$	2753, at $11\frac{1}{4}$ d.
4 d.	$\frac{1}{3}$	1376—6
1 d.	$\frac{1}{4}$	917—8
$\frac{1}{4}$ d.	$\frac{1}{4}$	229—5
		57— $4\frac{1}{4}$
	$\frac{1}{20}$	258 0— $11\frac{1}{4}$
		L. 129 0 $11\frac{1}{4}$

E X P L I C A T I O N.

In Ex. 1. I work first for 1 d.; which being $\frac{1}{12}$ s. I divide the given number by the denominator 12, and the quot is shillings, and the remainder pence: then, because 1 farthing is $\frac{1}{4}$ d. I divide the former quot by 4, and the sum of the quots is the price in shillings; which I divide by 20.

In Ex. 2. the rate $1\frac{1}{2}$ d. being an aliquot part of a shilling, the second method is shorter and better than the first.

In Ex. 3. I work first for $1\frac{1}{2}$ d. and then for $\frac{1}{4}$ d. by taking $\frac{1}{6}$ of the former quot.

In Ex. 7. I work first for 4 d. then I work for $\frac{1}{2}$ d.; which being $\frac{1}{8}$ of 4 d. I take $\frac{1}{8}$ of the first quot: lastly, I work for $\frac{1}{4}$ d.; which being $\frac{1}{2}$ of $\frac{1}{2}$ d. I take $\frac{1}{2}$ of the second quot; and, adding the three quots, I have the price in shillings.

In Ex. 8. I divide the rate into two parts, viz. 4 d. and $1\frac{1}{2}$; each of which being an aliquot part of a shilling, I take first $\frac{1}{3}$, and then $\frac{1}{8}$, of the given number, and add the quots.

In Ex. 9. I divide the 6 d. into two parts, viz. two 3 d. and then the $\frac{1}{4}$ d. is $\frac{1}{12}$ of 3 d. which makes an easy operation; but had I taken $\frac{1}{2}$ of the given number for 6 d. then $\frac{1}{4}$ d. would have been $\frac{1}{24}$ of 6 d. and 24 would have been a troublesome divisor.

Note 1. The above examples, and all others of the like

like kind, may also be solved by assuming fractions of a pound equivalent to the rate. Thus, in Ex. 6. the rate $3\frac{1}{2}$ d. is divided into 3 d. and $\frac{1}{2}$ d. and $3 \text{ d.} = \frac{1}{80} \text{ l.}$ and $\frac{1}{2} \text{ d.} = \frac{1}{6}$ of 3 d.; wherefore $3\frac{1}{2} \text{ d.} = \frac{1}{80} \text{ l.} + \frac{1}{6}$ of $\frac{1}{80} \text{ l.}$ Again, in Ex. 10 the rate $7\frac{3}{4}$ d. is divided into 4 d. 3 d. and $\frac{3}{4}$ d. and $4 \text{ d.} = \frac{1}{80} \text{ l.}$ and $3 \text{ d.} = \frac{1}{80} \text{ l.}$ and $\frac{3}{4} \text{ d.} = \frac{1}{4}$ of 3 d.; and so $7\frac{3}{4} \text{ d.} = \frac{1}{80} \text{ l.} + \frac{1}{80} \text{ l.} + \frac{1}{4}$ of $\frac{1}{80} \text{ l.}$ And these two examples wrought in this manner will stand as under.

Ex. 6.			Ex. 10.		
3 d.	$\frac{1}{80}$	347 1, at $3\frac{1}{2}$ d.	4 d.	$\frac{1}{80}$	52 0, at $7\frac{3}{4}$ d.
$\frac{1}{2}$ d.	$\frac{1}{6}$	43—7—9	3 d.	$\frac{1}{80}$	8—13—4
		7—4— $7\frac{1}{2}$	$\frac{3}{4}$ d.	$\frac{1}{4}$	6—10
		L. 50 12 $4\frac{1}{2}$			1—12—6
					L. 16 15 10

Note 2. Some, in working the above, or like examples, proceed in the following manner,

Ex. 6.

	s.	L.	s.	d.
3471, at 12 d. is	3471, or	173	11	
at 6 d. is		86	15	6
at 3 d. is		43	7	9
at $\frac{1}{2}$ d. is		7	4	$7\frac{1}{2}$
3471, at $3\frac{1}{2}$ d. is		L. 50	12	$4\frac{1}{2}$

Ex. 10.

	s.	L.	s.	d.
520, at 12 d. is	520, or	26		
at 6 d. is		13		
at 1 d. is		2	3	4
at $\frac{1}{2}$ d. is		1	1	8
at $\frac{1}{4}$ d. is		10	10	
520, at $7\frac{3}{4}$ d. is		L. 16	15	10

CASE

C A S E IV.

When the rate is shillings under twenty.

R U L E.

Multiply the given number by the numerators of the fractions contained in Tab. III. and divide the product by the denominators. Or, instead of this general rule, take the two particular ones following.

1. If the rate be an even number of shillings, multiply the given number by half the number of shillings in the rate, always doubling the right-hand figure of the product for shillings, and the rest are pounds.

2. If the rate be an odd number of shillings, work for the next lesser even number of shillings, as above; and for the odd shilling take $\frac{1}{20}$ of the given number.

E X A M P L E S.

1. When the rate is an even number of shillings.

Ex. 1.

436, at 2 s.

1

L. 43, 12 s.

Ex. 4.

48, at 8 s.

4

L. 19, 4 s.

Ex. 7.

326, at 14 s.

7

L. 228, 4 s.

Ex. 2.

127, at 4 s.

2

L. 25, 8 s.

Ex. 5.

87, at 10 s.

5

L. 43, 10 s.

Ex. 8.

48, at 16 s.

8

L. 38, 8 s.

Ex. 3.

56, at 6 s.

3

L. 16, 16 s.

Ex. 6.

420, at 12 s.

6

L. 252

Ex. 9.

52, at 18 s.

9

L. 46 16 s.

K k

2. When

2. When the rate is an odd number of shillings.

Ex. 10.

635, at 1s.

L. 31, 15s.

Ex. 11.

422, at 3s.

42 4

21 2

L. 63, 6s.

Ex. 12.

206, at 5s.

41 4

10 6

L. 51, 10s.

Ex. 13.

516, at 7s.

154 16

25 16

L. 180, 12s.

Ex. 14.

124, at 9s.

49 12

6 4

L. 55, 16s.

Ex. 15.

243, at 11s.

121 10

12 3

L. 133, 13s.

Ex. 16.

431, at 13s.

258 12

21 11

L. 280, 3s.

Ex. 17.

248, at 15s.

173 12

12 8

L. 186

Ex. 18.

324, at 17s.

259 4

16 4

L. 275, 8s.

Ex. 19.

425, at 19s.

382 10

21 5

L. 403, 15s.

Note 1. The reason of multiplying by half the number of shillings in the rate will appear by considering, that these are the numerators of the fractions denoting the rate. Thus, 2s. is $\frac{1}{10}$ l. and 4s. is $\frac{2}{10}$ l. and 6s. is $\frac{3}{10}$ l. and each unit in the product is two shillings. The division by the denominator 10 is performed by cutting off the right-hand figure of the product, and the figure so cut off is the remainder; and as each unit in the remainder is two shillings, the double of them is the remainder in shillings.

Note 2. From Ex. 1. we may learn, that when the rate

rate is 2 s. the price is found by doubling the right-hand figure of the given number for shillings, and the other figure or figures are pounds.

Note 3. In Ex. 2. the price may also be had by taking $\frac{1}{2}$ of the given number; and in this way every remainder will be 4 s.

Note 4. When the rate is 10 s. as in Ex. 5. the price may be obtained more easily by taking $\frac{1}{2}$ of the given number; and the remainder, if any, will be 10 s.

Note 5. When the rate is 5 s. as in Ex. 12. the price is more readily had by taking $\frac{1}{4}$ of the given number; and in this case every remainder is 5 s. In like manner, when the rate is 15 s. as in Ex. 17. the price may be found by taking $\frac{1}{2}$ of the given number, and then $\frac{1}{2}$ of that quot.

Note 6. By reversing the operation, from the price and any even rate given, we may readily find the quantity of goods, *viz.* Multiply the price by 10, that is, to the price annex a cipher, and divide the product by half the rate.

Ex. 1. How many yards, at 14 s. may be bought for 49 l. 7)490(70 yards. *Ans.*

Ex. 2. How many gallons, at 8 s. may be bought for 500 l.? 4)5000(1250 gallons. *Ans.*

C A S E V.

When the rate is shillings and pence, or shillings, pence, and farthings.

R U L E I.

If the rate be shillings and pence which make an aliquot part of a pound, divide the given number by the denominator of the fraction denoting the rate; the quot is pounds, and each unit of the remainder is equal to the rate.

E X A M P L E S.

Ex. 1.

$$\frac{1}{12} \left| \begin{array}{r} 354, \text{ at } 1 \text{ s } 8 \text{ d.} \\ \hline \text{L. } 29, 10 \text{ s.} \end{array} \right.$$

Ex. 2.

$$\frac{1}{8} \left| \begin{array}{r} 443, \text{ at } 2 \text{ s. } 6 \text{ d.} \\ \hline \text{L. } 55 \quad 7 \quad 6 \end{array} \right.$$

Ex. 3.

$$\frac{1}{6} \left| \begin{array}{r} 346, \text{ at } 3 \text{ s. } 4 \text{ d.} \\ \hline \text{L. } 57 \quad 13 \quad 4 \end{array} \right.$$

Ex. 4.

$$\frac{1}{3} \left| \begin{array}{r} 439, \text{ at } 6 \text{ s. } 8 \text{ d.} \\ \hline \text{L. } 146 \quad 6 \quad 8 \end{array} \right.$$

Ex. 5.

$$\frac{1}{3} \left| \begin{array}{r} 766, \text{ at } 13 \text{ s. } 4 \text{ d.} \\ \hline 255 \quad 6 \quad 8 \\ 255 \quad 6 \quad 8 \\ \hline \text{L. } 510 \quad 13 \quad 4 \end{array} \right.$$

Note. The fraction denoting the rate in Ex. 5. being $\frac{2}{3}$, you may also work thus: $766 \times 2 = 1532$, and $3)1532(510 \quad 13 \quad 4$.

R U L E II.

If the rate be no aliquot part of a pound, but may be divided into such parts, divide it accordingly, work for the parts separately, and then add.

E X A M P L E S.

Ex. 1.

$$\begin{array}{r} 427, \text{ at } 8 \text{ s.} \\ 6 \text{ d.} \\ \hline 128 \quad 2 \\ 53 \quad 7 \quad 6 \\ \hline \text{L. } 181 \quad 9 \quad 6 \end{array}$$

Ex. 2.

$$\begin{array}{r} 540, \text{ at } 5 \text{ s.} \\ 4 \text{ d.} \\ \hline 90 \\ 54 \\ \hline \text{L. } 144 \end{array}$$

Ex. 3.

$$\begin{array}{r} 386, \text{ at } 8 \text{ s.} \\ 8 \text{ d.} \\ \hline 128 \quad 13 \quad 4 \\ 38 \quad 12 \\ \hline \text{L. } 167 \quad 5 \quad 4 \end{array}$$

Ex. 4.

$$\begin{array}{r} 386, \text{ at } 14 \text{ s.} \\ 8 \text{ d.} \\ \hline 154 \quad 8 \\ 128 \quad 13 \quad 4 \\ \hline \text{L. } 283 \quad 1 \quad 4 \end{array}$$

Ex.

Ex. 5.				Ex. 6.			
796, at 3s. 10d.				394, at 17s. 4d.			
3s. 4d.	$\frac{1}{8}$	132	13 4	6s. 8d.	$\frac{1}{3}$	131	6 8
6d.	$\frac{1}{40}$	19	18	6s. 8d.	$\frac{1}{3}$	131	6 8
				4s.	$\frac{2}{10}$	78	16
		L. 132	11 4			L. 341	9 4

R U L E III.

If the rate be no aliquot part of a pound, and cannot readily be divided into such parts, divide it into parts whereof one at least may be an aliquot part of a pound, and the subsequent part, or parts, each an aliquot part of some prior part. Or,

Multiply the given number by the shillings, and then, for the pence and farthings, work as in Case III.

E X A M P L E S.

Method 1.				Method 2.			
Ex. 1.				Ex. 1.			
3506, at 1s. 3d.				3506, at 1s. 3d.			
1s.	$\frac{1}{20}$	175	6	1s.	$\frac{1}{4}$	876	6
3d.	$\frac{1}{4}$	43	16 6		$\frac{1}{20}$	438	2 6
		L. 219	2 6			L. 219	2 6
Ex. 2.				Ex. 2.			
95, at 1s. 10 $\frac{1}{2}$ d.				95, at 1s. 10 $\frac{1}{2}$ d.			
1s.	$\frac{1}{20}$	4	15	1s.	$\frac{1}{2}$	47	6
6d.	$\frac{1}{2}$	2	7 6	6d.	$\frac{1}{2}$	23	9
3d.	$\frac{1}{2}$	1	3 9	3d.	$\frac{1}{2}$	11	10 $\frac{1}{2}$
1 $\frac{1}{2}$ d.	$\frac{1}{2}$	11	10 $\frac{1}{2}$	1 $\frac{1}{2}$ d.	$\frac{1}{2}$	17	8 1 $\frac{1}{2}$
		L. 8	18 1 $\frac{1}{2}$		$\frac{1}{20}$	L. 8	18 1 $\frac{1}{2}$

Ex.

Method 1.

Ex. 3.

485, at 2 s. $2\frac{1}{4}$ d.

2 s.	$\frac{1}{10}$	48	10	
2 d.	$\frac{1}{12}$	4	00	10
$\frac{1}{4}$ d.	$\frac{1}{8}$		10	$1\frac{1}{4}$
L. 53 0 $11\frac{1}{4}$				

Ex. 4.

480, at 4 s. $8\frac{3}{4}$ d.

4 s.	$\frac{1}{5}$	96		
6 d.	$\frac{1}{8}$	12		
2 d.	$\frac{1}{3}$	4		
$\frac{1}{2}$ d.	$\frac{1}{4}$	1		
$\frac{1}{4}$ d.	$\frac{1}{2}$	0	10	
L. 113 10				

Ex. 5.

136, at 9 s. $2\frac{1}{2}$ d.

8 s.	$\frac{4}{10}$	54	8	
1 s.	$\frac{1}{20}$	6	16	
2 d.	$\frac{1}{6}$	1	2	8
$\frac{1}{2}$ d.	$\frac{1}{4}$		5	8
L. 62 12 4				

Method 2.

Ex. 3.

485, at 2 s. $2\frac{1}{4}$ d.

2 s.		970		
2 d.	$\frac{1}{12}$	80	10	
$\frac{1}{4}$ d.	$\frac{1}{8}$	10	$1\frac{1}{4}$	
$\frac{1}{20}$ 106 0 $11\frac{1}{4}$				

L. 53 0 $11\frac{1}{4}$

Ex. 4.

480, at 4 s.
 $8\frac{3}{4}$ d.

4 s.		1920		
6 d.	$\frac{1}{8}$	240		
2 d.	$\frac{1}{3}$	80		
$\frac{1}{2}$ d.	$\frac{1}{4}$	20		
$\frac{1}{4}$ d.	$\frac{1}{2}$	10		
$\frac{1}{20}$ 227 0				

L. 113 10

Ex. 5.

136, at 9 s. $2\frac{1}{2}$ d.

9 s.		1224		
2 d.	$\frac{1}{6}$	22	8	
$\frac{1}{2}$ d.	$\frac{1}{4}$	5	8	
$\frac{1}{20}$ 125 2 4				

L. 62 12 4

Ex.

Method 1.

Ex. 6.

370, at 14s. 2 $\frac{3}{4}$ d.

12 s.	$\frac{6}{10}$	222	
2 s.	$\frac{1}{10}$	37	
2 d.	$\frac{1}{12}$	3	1 8
$\frac{1}{2}$ d.	$\frac{1}{4}$	15	5
$\frac{1}{4}$ d.	$\frac{1}{2}$	7	8 $\frac{1}{2}$
		L. 263	4 9 $\frac{1}{2}$

Ex. 7.

1504, at 19 s. 9 d.

18 s.	$\frac{9}{10}$	1353	12
1 s.	$\frac{1}{20}$	75	4
6 d.	$\frac{1}{2}$	37	12
3 d.	$\frac{1}{2}$	18	16
		L. 1485	4

Method 2.

Ex. 6.

370, at 14 s. 2 $\frac{3}{4}$ d.

		148	
		37	
14 s.		5180	
2 d.	$\frac{1}{6}$	61	8
$\frac{1}{2}$ d.	$\frac{1}{4}$	15	5
$\frac{1}{4}$ d.	$\frac{1}{2}$	7	8 $\frac{1}{2}$
		526	4 9 $\frac{1}{2}$

L. 263 4 9 $\frac{1}{2}$

Ex. 7.

1504, at 19 s. 9 d.

		13536	
		1504	
19 s.		28576	
6 d.	$\frac{1}{2}$	752	
3 d.	$\frac{1}{2}$	376	
		2970	4
		L. 1485	4

Note 1. When the rate is more than 1 s. and less than 2 s. as in Ex. 1. and 2. there is no occasion, in working by Method 2. to draw a line under the given number: we esteem it so many shillings, and the parts for the pence or farthings are added up with it.

Note 2. In working the above or like examples, some people of inferior skill proceed in the following manner.

Ex.

Ex. 2. 95, at 1 s. 10½ d.

	L.	s.	d.
95 shillings make	4	15	
95 six-pences make	2	7	6
95 three-pences make	1	3	9
95 pence make		7	11
95 half-pence make		3	11½

Ans. 8 18 1½

C A S E VI.

When the rate is pounds.

R U L E.

Multiply the given number by the rate, and the product is the price in pounds.

E X A M P L E S.

Ex. 1.

42, at 2 l.

L. 84

Ex. 2.

13, at 8 l.

L. 104

Ex. 3.

30, at 3 l.

L. 90

Ex. 4.

48, at 12 l.

L. 576

C A S E VII.

When the rate is pounds and shillings, or pounds, shillings, pence, and farthings.

R U L E I.

If the rate be pounds and shillings, multiply the given number by the pounds, and work for the shillings as in Case IV.

EX-

EXAMPLES.

<i>Ex. 1.</i>		<i>Ex. 3.</i>	
1 l.	46, at 1 l. 4 s.	3 l.	58, at 3 l. 7 s.
4 s.	9 4	6 s.	174
		1 s.	17 8
	L. 55 4		2 18
	<i>Ex. 2.</i>		L. 194 6
	82, at 4 l. 10 s.		<i>Ex. 4.</i>
			26, at 3 l. 15 s.
4 l.	328	3 l.	78
10 s.	41	14 s.	18 4
		1 s.	1 6
	L. 369		
			L. 97 10

Note. When the rate is more than 1 l. and less than 2 l. as in *Ex. 1.* we have no occasion to draw a line under the given number, it being esteemed so many pounds, and the parts for the shillings or pence are added up with it.

R U L E II.

If the rate be pounds, with shillings and pence that make some aliquot part of a pound, or are divisible into aliquot parts, or into shillings and some aliquot part or parts; then multiply the given number by the pounds and work for the shillings and pence as in *Case V.* Rule I. or II.

EXAMPLES.

<i>Ex. 1.</i>		<i>Ex. 2.</i>	
	54, at L. 3 : 2 : 6.		43, at L. 5 : 3 : 4.
3 l.	162	5 l.	215
2 s. 6 d.	6 15	3 s. 4 d.	7 3 4
	L. 168 15		L. 222 3 4
			<i>Ex.</i>

L 1

Ex. 3.
 73, at L. 1:6:8.
 24 6 8

 L. 97 6 8

Ex. 4.
 76, at 7 l. 13 s.
 4 d.

 532
 25 6 8
 25 6 8

 L. 582 13 4

Ex. 5.
 83, at L. 2:8:6.

 2 l. 166
 6 s. 24 18
 2 s. 6 d. 10 7 6

 L. 201 5 6

Ex. 6.
 92, at L. 3:5:4.

 3 l. 276
 3 s. 4 d. 15 6 8
 2 s. 9 4

 L. 300 10 8

Ex. 7.
 37, at L. 4:8:8.

 4 l. 148
 6 s. 8 d. 12 6 8
 2 s. 3 14

 L. 164 0 8

Ex. 8.
 26, at L. 2:17:4.

 2 l. 52
 6 s. 8 d. 8 13 4
 6 s. 8 d. 8 13 4
 4 s. 5 4

 L. 74 10 8

R U L E III.

If the rate be pounds, with shillings, pence, and farthings, that cannot readily be resolved into aliquot parts of a pound; multiply the given number by the pounds; and then work for the shillings, pence, and farthings, as in Case V. Rule III. Method 1. But if you propose to work for the shillings, pence, and farthings, by Method 2. it will be convenient to do that in the first place, and then work for the pounds.

E X.

E X A M P L E S.

Method 1.

Ex. 1.

1l.	213, at 1l. 13s.	
		$4\frac{1}{2}$ d.
10s.	106	10
2s.	21	6
1s.	10	13
3d.	2	13 3
$1\frac{1}{2}$ d.	1	6 $7\frac{1}{2}$
	L. 355	8 $10\frac{1}{2}$

Ex. 2.

37, at 3l. 8s.
 $10\frac{1}{4}$ d.

3l.	111	
6s.	11	2
2s. 6d.	4	12 6
3d.		9 3
1d.		3 1
$\frac{1}{4}$ d.		$9\frac{1}{4}$
	L. 127	7 $7\frac{1}{4}$

Method 2.

Ex. 1.

213, at 1l. 13s.
 $4\frac{1}{2}$ d.

	639	
	213	
13s.	2769	
3d.	53	3
$1\frac{1}{2}$ d.	26	$7\frac{1}{2}$
	284	8 $10\frac{1}{2}$
	142	8 $10\frac{1}{2}$
	213	
	L. 355	8 $10\frac{1}{2}$

Ex. 2.

37, at 3l. 8s.
 $10\frac{1}{4}$ d.

6s.	222	
2s.	74	
6d.	18	6
3d.	9	3
1d.	3	1
$\frac{1}{4}$ d.		$9\frac{1}{4}$
	32	7 $7\frac{1}{4}$
	16	7 $7\frac{1}{4}$
	111	
	L. 127	7 $7\frac{1}{4}$

L 1 2

Method

Method 1.

Ex. 3.

416, at 2 l. 9 s.
3 $\frac{3}{4}$ d.

2 l.	832
8 s.	166 8
1 s.	20 16
3 d.	5 4
$\frac{3}{4}$ d.	1 6

L. 1025 14

Method 2.

Ex. 3.

416, at 2 l. 9 s.
3 $\frac{3}{4}$ d.

9 s.	3744
3 d.	104
$\frac{3}{4}$ d.	26
$\frac{1}{20}$	387 4
	193 14
2 l.	832

L. 1025 14

Note. A troublesome fraction in the rate may sometimes be avoided, by multiplying the rate, and dividing the quantity by the denominator of the fraction, and then computing the price of the quot from the new rate,

Ex. 1.

 $\frac{1}{8}$ 600, at 7 $\frac{7}{8}$ d.

75, at 5 s. 3 d.

5 s.	375
3 d.	18 9

 $\frac{1}{20}$ 39|3 9

L. 19 13 9

$$\begin{array}{rcl} d. & s. & d. \\ 7\frac{7}{8} \times 8 = 63 d. = & 5 & 3 \end{array}$$

Ex.

Ex. 2.

$\frac{1}{12}$	540, at $3\frac{1}{2}$ d.
	45, at 3 s. 11 d.
3 s.	135
4 d.	15
4 d.	15
3 d.	11 3
$\frac{1}{20}$	17 6 3
	L. 8 16 3

$3\frac{1}{2} \times 12 = 47 \text{ d.} = 3 \text{ 11}$

Ex. 3.

$\frac{1}{11}$	924, at 4 $5\frac{2}{11}$
	84, at L. 2 : 9 : 4
9 s.	756
4 d.	28
$\frac{1}{20}$	78 4
2 l.	39 4
	168
	L. 207 4

$4 \ 5\frac{2}{11} \times 11 = 2 \ 9 \ 4$

C A S E VIII.

When the given number consists of integers and parts.

R U L E.

Work for the price of the integers as already taught; and for the part or parts, take a proportional part, or parts, of the rate.

E X.

EXAMPLES.

Ex. 1.

Yards.	
720 $\frac{1}{4}$, at 6s. 8d. per yd.	
6s. 8d.	240
$\frac{1}{4}$ yd.	0 1 8
L. 240	1 8

Ex. 2.

Yards.	
116 $\frac{1}{2}$, at 4s. 6d. per yd.	
2s. 6d.	14 10
2s.	11 12
$\frac{1}{4}$ yd.	2 3
L. 26	4 3

Ex. 3.

Yards.
228 $\frac{3}{4}$, at 12s.
11d. p. yd.

12s.	27 36
4d.	76
4d.	76
3d.	57
$\frac{1}{2}$ yd.	6 5 $\frac{1}{2}$
$\frac{1}{4}$ yd.	3 2 $\frac{3}{4}$
$\frac{1}{20}$	295 4 8 $\frac{1}{4}$
L. 147	14 8 $\frac{1}{4}$

Ex. 4.

Yards.	
23 $\frac{7}{8}$, at 18s. per yd.	
18s.	20 14
$\frac{4}{8}$ yd.	9
$\frac{2}{8}$ yd.	4 6
$\frac{1}{8}$ yd.	2 3
L. 21	9 9

Ex. 5.

C. Q.	
365 2, at 5l. 5s. p. C.	
5l.	1825
5s.	91 5
2 Q.	2 12 6
L. 1918	17 6

Ex. 6.

C. Q. lb.	
28 3 14 at 1l. 10s. per C.	
1l.	14
10s.	0 15
2 Q.	7 6
1 Q.	3 9
14 lb.	
L. 43	6 3

Ex.

		<i>Ex. 7.</i>	
		<i>C. Q. lb.</i>	
		144 2 21, at 3l.	
		17s. 6d. p.C.	
3 l.	432		
15 s.	108		
2 s. 6d.	18		
2 Q.	1 18 9		
14 lb.	9 8 $\frac{1}{4}$		
7 lb.	4 10 $\frac{1}{8}$		
		L. 560 13 3 $\frac{3}{8}$	
		<i>Ex. 8.</i>	
		<i>T. C. Q.</i>	
		73 17 2, at 9l.	
		12s. 10d.	
		per tun.	
9 l.	657		
10 s.	36 10		
2 s.	7 6		
6 d.	1 16 6		
2 d.	12 2		
2 d.	12 2		
10 C.	4 16 5		
5 C.	2 8 2 $\frac{1}{2}$		
2 C. 2 Q.	1 4 1 $\frac{1}{4}$		
		L. 712 5 6 $\frac{3}{4}$	

		<i>Ex. 9.</i>	
		<i>lb. Tr. oz. dw.</i>	
		36 8 16, at	
		23 l. 16 s.	
		4 $\frac{1}{2}$ d. per lb.	
		108	
		72	
23 lb.	828		
10 s.	18		
5 s.	9		
1 s.	1 16		
4 d.	12		
$\frac{1}{2}$ d.	1 6		
4 oz.	7 18 9 $\frac{1}{2}$		
4 oz.	7 18 9 $\frac{1}{2}$		
16 dw.	1 11 9 $\frac{1}{10}$		
		L. 874 18 10 $\frac{1}{10}$	

An operation in the rules of practice may be proved by running over the several steps a second time, by working the same question a different way, or by the rule of three.

Practical Questions.

- | | |
|-------------------------------------|-----------------------------------|
| 1. 126, at 14 $\frac{3}{4}$ d. | <i>L. s. d.</i> |
| | <i>Ans.</i> 7 14 10 $\frac{1}{2}$ |
| 2. 478, at 4 s. 10 $\frac{1}{2}$ d. | <i>Ans.</i> 116 10 3 |
| | 3. 50, |

3. 50, at L. 3 : 15 : $6\frac{1}{2}$ *Ans.* L. 188 17 1
4. $419\frac{1}{2}$ yards, at 4s. $10\frac{1}{4}$ d. p. yd. *Ans.* L. 101 16 $3\frac{7}{8}$ f.
5. 75 C. 2 Q. 14 lb. at 15s. $4\frac{1}{2}$ d. per C. *Ans.* L. 58 2 8 $3\frac{1}{4}$
6. 8 T. 15 C. 3 Q. 27 lb. at 14l. per tun. *Ans.* L. 123 3 $10\frac{1}{2}$

7. A bankrupt's effects amount to 811l. 10s.; and he owes

	L.	s.	d.
To A	220	16	6
To B	312		
To C	117	12	6
To D	106	12	6
To E	200	6	
To F	124	12	6

In all 1082

How much can he afford to pay *per* pound, and what must each creditor have?

Ans. 15s. *per* pound; and the creditors get as follows:

	L.	s.	d.
A	165	12	$4\frac{1}{2}$
B	234		
C	88	4	$4\frac{1}{2}$
D	79	19	$4\frac{1}{2}$
E	150	4	6
F	93	9	$4\frac{1}{2}$

Total 811 10



The End of the First Volume.

